Influence of Parameters on the Performance of a MOACO Algorithm for Solving the Bi-Criteria Military Path-finding Problem

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***Abstract*— This paper presents a statistical parameter analysis of the ant colony optimization algorithm that was implemented to solve the bi-criteria military path-finding problem. Three parameters have been studied using analysis of variance (ANOVA) in order to identify their influence in the results and the most suitable values for them: number of ants, number of iterations and exploration/exploitation factor. In addition, a mean analysis has been performed in order to complete the conclusions obtained. The study has yielded optimal values for the parameters under study, and some internal relationships between them have been identified.**

I. Introduction

Many metaheuristic or optimization algorithms need some parameters to be set in order to obtain good solutions. Usually, those values are 'calculated' in an empirical (or heuristical) way. However, the best way is to apply some statistical methods to obtain them and, in addition, a detailed statistical analysis of the influence of every parameter has to be made, so that the designer should pay most attention to the parameter presenting values that yield the statistically most significant performance changes.

In order to determine the most important parameters, and to establish the most suitable values for such parameters (thus obtaining an optimal operation), the ANOVA (ANalysis Of the VAriance) [1] method has been used in this work. This statistical tool, based on the analysis of the mean variance, is widely used to obtain the significance and relative importance of the parameters with respect to results, as well as suitable values for them [2].

This paper extends previous research on the *military unit path-finding problem*, which could be defined as finding the best path for a military unit, from an origin to a destination point in the battlefield, keeping a balance between route speed and safety, considering the presence of enemies (which can fire against the unit) and taking into account realistic properties and restrictions. Both the speed and the safety of the path are the two main criteria that a commander should take into account inside a battlefield when the unit he leads must to accomplish a mission (i.e. reach a target place).

To solve this problem we designed a Multi-Objective Ant Colony Optimization Algorithm (MOACO [3]), named hCHAC (presented in [4], [5], [6]). It is an Ant Colony System, a type of ACO [7], [8] which allows to control the balance between search exploration and exploitation. This algorithm has been adapted to deal with two objectives (see [9] for an overview of multi-objective optimization).

Thus, in this work, we use statistical analysis and methods such as ANOVA, to search for the best set of parameter values for the hCHAC algorithm. The rest of the paper is structured as follows: A brief problem and hCHAC algorithm description is shown in section II. The parameters to study are presented in section III. Section IV contains the basis of the statistical method ANOVA. The statistical analysis is commented in section V (experimental setup and conclusions) and finally, in section VI the conclusions and the future work in this line are exposed.

II. Problem and Methodology

A. Problem Definition

The problem is modelled considering that the unit has a *level of energy (health)* and a *level of resources*, which are consumed when it moves along the path, so the problem objectives are adapted to minimize resource and energy consumption. The battlefield is modelled as a grid of hexagonal cells with a *cost in resources*, which represents the difficulty of going through it, and a penalization and a *cost in energy*, which means the unit depletes its human resources or vehicles suffer damage when crossing over the cell (*no combat casualties*). Both costs depend on the cell type. Besides, moving between cells with different heights also costs resources (more if it goes up), and falling in a weapons impact zone depletes energy. Figure 1 shows an example of real world battlefield and the information layer associated to it, which has been created using a custom-made application.

B. hCHAC Features

hCHAC means *Compañía de Hormigas ACorazadas* (Armored Ant Company) with the prefix 'hexa' due to the grid topology [4]. It is an Ant Colony System (ACS) [8] adapted to deal with several objectives, that is, a Multi-objective Ant Colony Optimization algorithm (MOACO) [9], [3]. In these algorithms, the problem is transformed into a graph where each node corresponds to a cell in the map and an edge between two nodes is the connection between neighbor cells in the map. Every edge has two weights associated which are the costs in resources and energy that going through that edge causes to the unit.

In every iteration, the ants separately build a complete path (solution), between origin and destination points (if possible), by travelling through the graph. To guide this movement they use a State Transition Rule (STR) which combines two kinds of information: pheromone trails (learnt information) and

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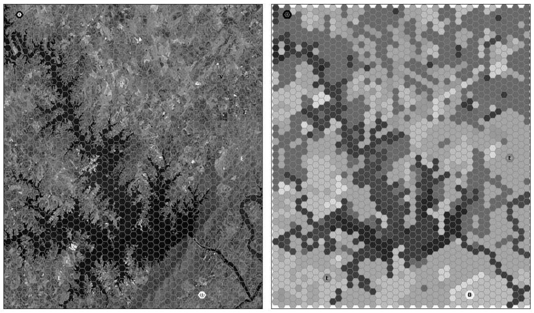

Fig. 1. Example Map (45x45 cells). The image on the right-hand side is a real world picture showing a lake surrounded by some hills and lots of vegetation. On the left-hand side it is shown its associated information layer, where it can be seen the types corresponding to the same hexagons in the other image. The different shades in the same color models height (light color) and depth (dark color). There are two enemies labelled with 'E', an origin point (in the top-left corner of the images) labelled with 'O' and a destination point (in the bottom-right) labelled with 'D'. These labels are black on the image at right and white on the left.

heuristic knowledge. ACSs are used to have better control in the balance between exploration and exploitation by using the characteristic parameter q_0.

As previously said, the problem has two independent objectives to minimize. These objectives are named ***f***, minimization of the resources consumed in the path (fast path or speed maximization) and ***s***, minimization of the energy consumed in the path (safe path or safety maximization).

hCHAC uses two pheromone matrices (τ_f,τ_s) and two heuristic functions (η_f,η_s) (one per objective), a single colony, and two STRs: (*Combined State Transition Rule, CSTR*), similar to the one proposed in [10] and (*Dominance State Transition Rule, DSTR*), which ranks neighboring cells according to how many (of the neighbors) they dominate. These rules use the parameter $\lambda \in (0,1)$, which is user-defined, and sets the importance of the objectives in the search (which one has higher priority and how much). If the user decides to search for a fast path, λ will take a value close to 1, on the other hand, if he wants a safe path, it has to be close to 0. This value is constant during the algorithm for all ants, so *hCHAC* searches always in the same zone of the space of solutions (the zone related to the chosen value for λ). The local and global pheromone updating formulae are based in the MACS-VRPTW algorithm proposed in [11], [12], with some changes due to the use of two pheromone matrices. Finally, there are two evaluation functions (used to assign a global cost value to every solution found) named F_f (minimization of resources consumption) and F_s (minimization of energy consumption).

There is one *Heuristic Function* per objective which try to guide the search considering the key factors for each objective. They assign a value to every edge in the graph that includes the heuristic knowledge of the problem, so that for edge *(i,j)* they are:

$$\eta_f(i,j) = \frac{\omega_f^f}{R(i,j)} + \frac{\omega_f^d}{d(j,T)} + \left(\omega_f^h \cdot H(j)\right) \tag{1}$$

$$\eta_s(i,j) = \frac{\omega_s^s}{E(i,j)} + \frac{\omega_s^d}{d(j,T)} + \left(\omega_s^h \cdot H(j)\right) \tag{2}$$

In Equation 1, *R(i,j)* is the cost in resources when moving from node *i* to node *j*, *d* is the Euclidean distance between two nodes (*T* is the target node of the problem) and *H* is the visibility of a cell, which is a score (between 0 and 1), being 1 when the cell is hidden to all the enemies (or to all the cells in a radius when there are no enemies) and decreasing exponentially when it is not (it depends on the number of enemies or cells in a radius, if there are no enemies, which can see the cell). ω_f^f , ω_f^d and ω_f^h are weights to assign relative importance to the terms in the formula. In this case, the most important term is the distance to target point because, when searching for the fastest path, a straight path will be better. The cost in resources is also important, but less so; and finally the visibility has a small influence, because it is disregarded almost completely in the case of trying to follow the fastest path.

In Equation 2, *E(i,j)* is the cost in energy of moving from node *i* to node *j* (but it only depends on j), *d* and *H* are the same as the previous formula. ω_s^s, ω_s^d and ω_s^h are again weights to assign relative importance to the terms in the formula, but in this case the main factor is visibility, following by cost in energy (both are to be considered in a safe path), and a little the distance to target point.

The *Combined State Transition Rule* (CSTR) is similar to the pseudo-random-proportional rule used in ACS, but adapted to deal with a two objectives problem by combining the heuristic and pheromone information of both of them (Equations 3 and 4).

In that rule, $q_0 \in [0,1]$ is the standard ACS parameter and q is a random value in [0,1]. τ_f, τ_s and η_f, η_s as well as the λ parameter are the previously commented. α and β are the usual (in ACO algorithms) weighting parameters for pheromone and heuristic information respectively, and N_i is the current feasible neighborhood for the node *i*.

This state transition rule works as follows: when an ant is building a solution path and is placed at one node *i*, a random number *q* in [0,1] is generated, if $q \leq q_0$ the best neighbor *j* is selected as the next node in the path (Equation 3). Otherwise, the algorithm decides which node is the next by using a roulette wheel considering *P(i,j)* as probability for every feasible neighbor *j* (Equation 4).

The other implemented rule is the *Dominance State Transition Rule* (DSTR), which is based on the dominance concept of multi-objective problems (see reference [9]). It is defined as follows (*a* dominates *b*):

$$\begin{aligned} &a \prec b \;\; if: \\ &\forall i \in 1,2,...,k \mid C_i(a) \leq C_i(b) \quad \wedge \quad \exists j \in 1,2,...k \mid C_j(a) < C_j(b) \end{aligned} \tag{5}$$

where *a* and *b* are two different vectors of *k* values (one per objective) and *C* is a cost function for every component in the vector. If it intends to minimize the cost and Equation 5 is true, then *b* is dominated by *a*.

Therefore, in our problem there are two cost functions to evaluate the dominance between edges because they have assigned pheromone and heuristic information, which are

If $(q \leq q_0)$

$$j = \arg\max_{j \in N_i} \left\{ \tau_f(i,j)^{\alpha \cdot \lambda} \cdot \tau_s(i,j)^{\alpha \cdot (1-\lambda)} \cdot \eta_f(i,j)^{\beta \cdot \lambda} \cdot \eta_s(i,j)^{\beta \cdot (1-\lambda)} \right\} \quad (3)$$

Else

$$P(i,j) = \begin{cases} \dfrac{\tau_f(i,j)^{\alpha \cdot \lambda} \cdot \tau_s(i,j)^{\alpha \cdot (1-\lambda)} \cdot \eta_f(i,j)^{\beta \cdot \lambda} \cdot \eta_s(i,j)^{\beta \cdot (1-\lambda)}}{\sum_{u \in N_i} \tau_f(i,u)^{\alpha \cdot \lambda} \cdot \tau_s(i,u)^{\alpha \cdot (1-\lambda)} \cdot \eta_f(i,u)^{\beta \cdot \lambda} \cdot \eta_s(i,u)^{\beta \cdot (1-\lambda)}} & if\ j \in N_i \\ 0 & otherwise \end{cases} \quad (4)$$

combined in each function using the same parameters as in CSTR formulae (Equations 3 and 4).

$$C_f(i,j) = \tau_f(i,j)^{\alpha \cdot \lambda} \cdot \eta_f(i,j)^{\beta \cdot \lambda} \quad (6)$$

$$C_s(i,j) = \tau_s(i,j)^{\alpha \cdot (1-\lambda)} \cdot \eta_s(i,j)^{\beta \cdot (1-\lambda)} \quad (7)$$

In addition, there is a function which uses the concept presented in Equation 5 (which uses Equations 6 and 7 as cost functions):

$$D(i,j,u) = \begin{cases} 1 & if\ (i,j) \prec (i,u) \\ 0 & otherwise \end{cases} \quad (8)$$

Finally, the dominance state transition rule is showed in Equations 9 and 10. In this rule, N_i is again the current feasible neighborhood for the node *i*. The rule chooses the next node *j* in the path (when an ant is placed at node *i*) considering the number of neighbors dominated for every one. So it works like the previous CSTR, but taking into account a dominance-based criteria for the max or the probability roulette wheel.

As previously said, this rule applies multi-objective problem concepts which allow to compare nodes without using an aggregative expression which combines information of both objectives. The DSTR tends to be more 'exploratory' (usually there will be some nodes with the same value for P), so it is necessary to use parameter values which tends to exploitation in order to balance the search.

There are two *Evaluation Functions* (one per objective, again) which are used to assign a global cost value to every solution found by each ant. They consider not only the cost in resources or energy of every edge in the path, but the visibility of the cells too:

$$F_f(S_p) = \sum_{n \in S_p} [R(n-1,n) + \omega^h_{F,f} \cdot (1 - H(n))] \quad (11)$$

$$F_s(S_p) = \sum_{n \in S_p} [E(n-1,n) + \omega^h_{F,s} \cdot (1 - H(n))] \quad (12)$$

where S_p is the solution path to evaluate and *n* is a node in that path. $\omega^h_{F,r}$ and $\omega^h_{F,e}$ are weights related to the importance of visibility of the cells in the path. In Equation 11 its importance will be small, since it is less important to hide in a fast path; and it will be high in Equation 12 for the opposite reason. The other terms are the same as in Equations 1, 2.

Since hCHAC is an ACS, there are two levels of pheromone updating, local and global, which update two matrices at each level. The equations for *Local Pheromone Updating* (performed when a new node *j* is added to the path that an ant is building) at time t are:

$$\tau^t_x(i,j) = (1-\rho) \cdot \tau^{t-1}_x(i,j) + \rho \cdot \tau_{0,x} \quad (13)$$

where $x = f, s$, ρ in [0,1] is the common evaporation factor and $\tau_{0,x}$, $\tau_{0,s}$ are the initial amounts of pheromone in every edge for every objective, respectively:

$$\tau_{0,f} = \frac{1}{(n_c \cdot M_R)} \quad (14)$$

$$\tau_{0,s} = \frac{1}{(n_c \cdot M_E)} \quad (15)$$

with n_c as the number of cells in the map to solve, M_R as the maximum amount of resources going through a cell may require, and M_E as the maximum cost in energy going through a cell may produce (in the worst case).

The equations for *Global Pheromone Updating* at time t are:

$$\tau^t_x(i,j) = (1-\rho) \cdot \tau^{t-1}_x(i,j) + \rho / F_x \quad (16)$$

where $x = f, s$ again. Only the solutions inside the Pareto set (non-dominated solutions) will be updated when all ants have finished building paths in every iteration. This update depends on the cost of the solution found by the ant, given by the evaluation functions (Equations 11 and 12).

III. Parameters to Consider

Subsection (II-B) shows the formulae in which the parameters of the algorithm take part: α and β weight the terms in the state transition rules, ρ which is used in the pheromone update formulae, q_0, the typical proportion factor of ACS algorithms, and λ a user defined value.

Their influence on the algorithm behaviour (from the search point of view) is:

- α: sets the importance of the pheromone trails (high value means higher exploration component).
- β: sets the importance of the heuristic information (high value means higher exploitation component).
- λ: sets the priority of each one of the two objectives.
- ρ: sets the evaporation rate of the trails (high values means more exploration).
- q_0: controls the balance between exploration and exploitation in the search.

Many works have analyzed the best values for the basic parameters in traditional ACO algorithms (α, β and ρ), from

If $(q \leq q_0)$

$$j = \arg\max_{j \in N_i} \left\{ \sum_{u \in N_i} D(i,j,u) \quad \forall j \neq u \right\} \tag{9}$$

Else

$$P(i,j) = \begin{cases} \dfrac{\left(\sum_{u \in N_i} D(i,j,u)\right) + 1}{\sum_{k \in N_i} \left(\left(\sum_{u \in N_i} D(i,k,u)\right) + 1 \right)} & if\, j \in N_i \wedge j \neq u \wedge k \neq u \\ 0 \quad otherwise \end{cases} \tag{10}$$

Dorigo *et al.* [13] to more recent works [14]. All of them are in agreement in using α**=1,** β**=2** and ρ**=0.1** as default values, so we also consider those values in our experiments.

On the other hand, λ is a new parameter introduced in some bi-criteria ACOs (as hCHAC is) to set the importance of one objective over the other, so its influence is well known [15] (and chosen by the user).

The last parameter (q_0) is the only one (of the previously commented) to be considered in this study, because its value can drive the search by introducing a high exploration level (search in many different zones of the space of solutions, but yielding worse solutions), or considering a high exploitation level (improving the solutions, but in a smaller zone of the space), besides it is very flexible, and so, less predictable to set a good value for it. In addition, there are two extra parameters to take into account in an ACO algorithm: the *Number of Iterations* (N_{Its}), which usually introduces an extra exploitation component (the solutions can be improved more times), and the *Number of Ants* (N_{Ants}), which corresponds to an increasing in the exploration component (they can explore more areas of the space of solutions). So these are the three parameters to be analyzed in this paper.

IV. The Analysis of Variance

The theory and methodology of ANOVA was mainly developed by R.A. Fisher during the 1920s [1], [16]. ANOVA examines the effects of one or several quantitative or qualitative variables (called factors) on one quantitative response, and is applied when the relation between factors and response needs to be assessed. It is essentially a method of analyzing the variance to which a response is subject, dividing it into the various components corresponding to the sources of variation, which can be identified.

The ANOVA method allows us to determine whether a change in the responses is due to a change in a factor or due to a random effect. Thus it is possible to determine the variables with greatest effect on the method that is being evaluated. The basic assumptions made to apply this statistics tool satisfactorily [17] are that the observations obtained should be mutually independent, distributed according to a normal distribution, have the same variance (σ^2), and averages that can be expressed as a linear combination of certain unknown parameters.

With ANOVA, it is tested a null hypothesis which considers that all of the population means are equal against the alternative hypothesis that there is at least one mean that is not equal to the others. Once the sample mean and variance for each level (value) of the main factor are founded, and using these values, two different estimations of the population variance are yielded. The first one is obtained by finding the sample variance of the n_k sample means from the overall mean. This variance is referred to as the *variance among the means*. The second estimation of the population variance is found by using a weighted average of the sample variances. This variance is called the *variance within the means*.

The estimations which ANOVA offers are based on the value of statistical *F*:

$$F = \frac{S_T}{S_R} \tag{17}$$

where S_T is the sum of the squares of the observations according to the levels (values) of all the factors and S_R is the sum of the squares according to each level, both of which are divided by the number of degrees of freedom (*DF*). *F* is compared with the F-Snedecor distribution [18] with the appropriate number of degrees of freedom, to obtain a significance value (*Sig. Level*). If this level is lower than 0.05, then the influence of the factor is statistically significant at the confidence level of 95%.

Besides the ANOVA method, an analysis of the mean values is performed to decide the most suitable values for each parameter. In those cases where evaluating the difference of means between two groups is necessary, statistical *t-Student* tests are used, as these can be used even on small samples. This method calculates a value *p* that represents the error probability if the null hypothesis is accepted, that is, the error probability supposing that there is no difference between the levels of the observations in the population.

V. Statistical Analysis of hCHAC

A. Experimental Setup

In this section, the ANOVA statistical tool is applied to determine whether the influence of a change in the parameter values (levels) is significant in the costs of the solutions yielded, to establish the most suitable values for these parameters (in order to obtain the best solutions as possible), and to design a general set which yields good results in maps with different configurations and associated difficulty.

hCHAC has been used to find the best path in several maps [15], [4], [5], obtaining the following values heuristically: α=1, β=2, ρ=0.1, and q_0=0.4, which are their usual values except for q_0, which usually takes the value 0.1; in our case, that makes the exploration component higher. The value for λ is usually chosen by the user, in order to set the priority of each one of the objectives. In this work it has been set as **λ=0.5**, so both objectives have the same relevance.

A 1-Factor ANOVA have been applied, considering as factor the *configuration set of parameters*, taking into account as levels all the possible combinations of values of the three parameters to analyze. They are shown in Table I.

The study includes two algorithms/methods: hCHAC using the Combined State Transition Rule (CSTR) and hCHAC using the Dominance-Based State Transition Rule (DSTR) on two maps:

- *PG Map*: realistic Map based on an scenario of the Panzer General^{TM}game modelled using our application. There are two enemies watching and firing against the unit.
- *VM Map*: map designed for training where there are mountains, valleys and two enemy units on watch.

Each method have been run 15 times, in each map and considering every one of the parameter configurations (levels of the factor). Several dependent variables have been extracted from the Pareto set as objects of the ANOVA method:

- F_f/F_s *Fast*: the cost in resources/energy corresponding to the fastest solution (the solution with the smallest cost in resources).
- F_f/F_s *Safe*: the cost in resources/energy corresponding to the safest solution (the solution with the smallest cost in energy).
- F_f/F_s *Mean*: the average cost in resources/energy of all the solutions in the Pareto Set.
- $SolsPS$: the number of solutions in the Pareto Set.

TABLE I
LEVELS OF THE FACTOR OF ANOVA

Level	q_0	N_{Its}	N_{Ants}	Level	q_0	N_{Its}	N_{Ants}
1	0.1	800	20	16	0.4	2200	20
2	0.1	800	50	17	0.4	2200	50
3	0.1	800	80	18	0.4	2200	80
4	0.1	1500	20	19	0.8	800	20
5	0.1	1500	50	20	0.8	800	50
6	0.1	1500	80	21	0.8	800	80
7	0.1	2200	20	22	0.8	1500	20
8	0.1	2200	50	23	0.8	1500	50
9	0.1	2200	80	24	0.8	1500	80
10	0.4	800	20	25	0.8	2200	20
11	0.4	800	50	26	0.8	2200	50
12	0.4	800	80	27	0.8	2200	80
13	0.4	1500	20				
14	0.4	1500	50				
15	0.4	1500	80				

B. Results of the Statistical Analysis

The results of apply the 1-Factor ANOVA to the sets of solutions yielded by each method (CSTR and DSTR), in each map, are considered to determine their importance and the most suitable parameter value. The tables shown are those given by the SPSS (v.14) program.

The ANOVA tables show, for each source of the experiment, the number of degrees of freedom (DF), the sum of squares (SS), the mean of the squares (MS), the value of the statistical F and its significance level (Sig.); if the latter is smaller than 0.05, then the factor effect is statistically significant at the level of confidence of 95%, but we also consider that the value of F must be greater than 4.5-5 as a criteria of relevance. These significant factors and their significance levels are highlighted in boldface in the tables. We also support our analysis by using Figure 2, where the means for the experiments made using all the levels of the configuration factor are shown.

In spite of the algorithm yields multi-objective solutions, we analyze them separately in order to apply a 1-Factor ANOVA. But, since the two objectives are independent, usually a good fast path corresponds to a very unsafe one and a safe path to a slow route. This is the reason why the values are so different in the tables, so it can be seen that usually the values for the cost in energy (F_s) are very high in the search for fast paths and the other way round.

Firstly, Table II (top) contains the results of ANOVA when considering the values yielded for the runs of the experiments, applying the CSTR method, and using the 27 levels of factor in the PG map. In spite of all the significance levels are lower than 0.05, there is no value for F greater than 5, and only for F_fMean is greater than 4.5. So it is the source which has had more variation in its results relating to the rest of the sources. We made an incremental 1-Factor ANOVA study in order to find the configuration (the level) which produces the greatest variation between the different levels of the factor. It consist on several consecutive ANOVA tests applied to a set of levels, beginning with the two first

TABLE II
PG MAP - CSTR (TOP) AND DSTR (BOTTOM) METHODS - ANOVA RESULTS. SUM OF SQUARES (SS), NUMBER OF DEGREES OF FREEDOM (DF), MEAN OF THE SQUARES (MS), VALUE OF THE STATISTICAL F AND ITS SIGNIFICANCE LEVEL (SIG.) FOR EACH SOURCE.

Source	SS	DF	MS	F	Sig.
F_fFast	1969.27	26	65.24	4.2	0
F_sFast	315832.91	26	12147.42	2.97	0
F_fSafe	2641.144	26	101.58	2.22	0
F_sSafe	315832.91	26	12147.42	2.97	0
F_fMean	1620.57	26	62.33	**4.72**	**0**
F_sMean	135681.72	26	5218.53	3.54	0
$SolsPS$	243.04	26	9.35	1.83	0.01

Source	SS	DF	MS	F	Sig.
F_fFast	5377.74	26	206.84	**20.77**	**0**
F_sFast	529753.74	26	20375.14	**10.87**	**0**
F_fSafe	59292.73	26	2280.49	**26.86**	**0**
F_sSafe	3062.57	26	117.79	**16.51**	**0**
F_fMean	23193.24	26	892.05	**46.57**	**0**
F_sMean	91222.74	26	3508.57	4.09	0
$SolsPS$	66.4	26	2.55	1.41	0.09

and adding one level each time (incrementing the degrees of freedom). If there is a great variation in the results of the test relating to the preceding trial (high value of F), we consider that the last level of the factor is the most relevant. So, in our incremental test, we found that the great variations appears when we consider the last levels: 24,26 and 27, being the greatest with the last one (F=4.7). This means that the configuration: q_0=0.8, N_{Its}=2200 and N_{Ants}=80, offers the results with the highest variation relating to the rest of results for $F_f Mean$.

In order to know if this set of values for the parameters is good, we can look at Figure 2 (top-left) in the correspondent values (the $F_f Mean$ line) and notice that this level of the factor yields the best solutions (in mean). Second, we study the other state transition rule (DSTR method) on the same map; results are shown in Table II (bottom). This time most sources obtain a high value of F, with a significance level of 0. So we made again the incremental test, but considering all these sources ($F_f Fast$,$F_s Fast$,$F_f Safe$,$F_s Safe$ and $F_f Mean$). In the test using the two first levels of the factor, all the sources related to F_s have a high variation, but this is not too relevant. However, there is a turning point at level 12, where the variations begin to be significant for $F_s Safe$ (F=8.6) and $F_f Mean$ (F=6.63). The big gap in variations appears when level 19 is included: all sources initially commented take a high F value, being the highest, the last one, corresponding again to the level 27. At level 19 and above q_0=0.8, as seen in (Table I). If we look again at Figure 2 (bottom-left), we realize that all the best mean values (for all the sources) correspond to that value (the $q0$ values are separated by vertical lines).

TABLE III

VM Map - CSTR (top) and DSTR (bottom) methods - ANOVA Results. Sum of squares (SS), number of degrees of freedom (DF), mean of the squares (MS), value of the statistical F and its significance level (Sig.) for each source.

Source	SS	DF	MS	F	Sig.
$F_f Fast$	935.76	26	35.99	4.02	0
$F_s Fast$	60280.98	26	2318.5	3.61	0
$F_f Safe$	242.48	26	9.33	1.49	0.06
$F_s Safe$	1463.53	26	56.29	2.86	0
$F_f Mean$	341	26	13.12	2.62	0
$F_s Mean$	14037.78	26	539.91	3.12	0
$SolsPS$	27.98	26	1.08	4.12	0

Source	SS	DF	MS	F	Sig.
$F_f Fast$	4398.48	26	169.17	**19.8**	**0**
$F_s Fast$	88656.44	26	3409.86	**9.9**	**0**
$F_f Safe$	21643.78	26	832.45	**17.39**	**0**
$F_s Safe$	108870.87	26	4187.34	**23.18**	**0**
$F_f Mean$	10686.67	26	411.03	**30.24**	**0**
$F_s Mean$	98576.14	26	3791.39	**28.35**	**0**
$SolsPS$	70.84	26	2.73	3.89	0.09

The same analysis has been made in the VM map, but in this case, there is no interesting results when the algorithm use the CSTR method (see Table III, up), because no one of the statistical F's take an enough high value, which means there is not great variations in the results of the experiments. Finally, the test on the VM map using the DSTR method shows (in Table III, bottom) high values for F (with a significance level of 0) for all sources, except for $Sols_{PS}$. The incremental study highlights that the greatest variations arise when we introduce levels 19 to 27, being the highest in the last level. Figure 2 (bottom-right) also shows better solutions in that zone (note also the last configuration yields the best results), which in turn implies that the most appropriate value for q_0 is 0.8 in this case.

Once the ANOVA method has been applied and interpreted, an analysis of the mean values is performed, by researching the values of mean and standard deviation for all the experiments in the two maps (Figure 2, results for $SolsPS$ have been omitted). The best values marked in the graphs have been verified using *t-Student* statistical tests by comparing the best value for each source (black square) with the fourth best (not marked) and with the worse of the means. Significant differences were found at the confidence level of 80% in most the cases between the best and the fourth best (sometimes 90% or 95%) and always at the confidence level of 95%-99% in the comparison between the best and the worst, which means that differences are relevant.

Figure 2 for the PG map shows that four of the sources ($F_f Fast$, $F_f Safe$, $F_s Safe$ and $F_f Mean$) take their lowest values in the area correspondent to levels 19 to 27 in CSTR (top-left), and all of them in the case of DSTR (bottom-left). This area is related to the value 0.8 for the parameter q_0. Specifically, the best values (black squares) are most of times yielded for the last configuration. The exceptions occurs in some of the cases related to the safety objective ($F_s Fast$ and $F_s Mean$ cost), when the best solution is not chosen depending on the value of F_s, so they have less relevance for the conclusions of the work.

If we consider the two other parameters of the study: N_{Its} should take higher values to get better results (it is reasonable). The exceptions occur in the safety objective (F_s), because it has not been assigned the highest priority. On the other hand, N_{Ants} is more difficult to analyze, because increasing the number of ants means a higher exploration component, which works great if q_0 takes high values (promotes the exploitation). The exception appears when q_0 takes a low value (0.1), where sometimes increasing the number of ants yields a worse solution. The reasons for this are explained in section V-C.

As seen in Figure 2 for the VM map, the situation is similar to the previous one. Four of the sources ($F_f Fast$, $F_f Safe$, $F_s Safe$ and $F_f Mean$) take their best values in levels 19 to 27 in CSTR (q_0=0.8) and $F_s Fast$ and $F_s Mean$ for q_0=0.1. Regarding the values for the two other parameters, the situation is the same as in the other map. Again, *t-Student* statistical tests have been applied, getting the same results as before (the differences between the results of the experiments are anew relevant).

C. Conclusions of the Statistical Analysis

The most interesting configuration supported by the ANOVA tests results corresponds to factor level 27: q_0=0.8, N_{Its}=2200, N_{Ants}=80, but this should be qualified by doing an analysis of the values of means (Figure 2). Those values

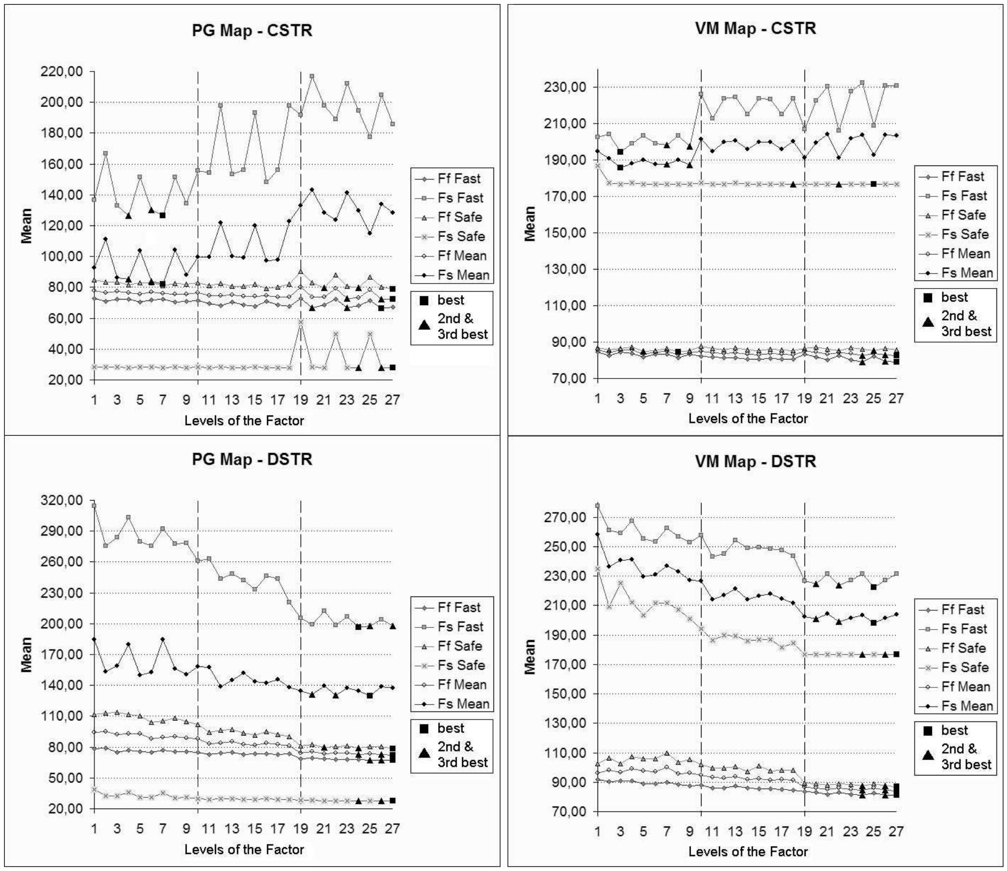

Fig. 2. Mean values for all the experiments considering each source for CSTR (top) and DSTR (bottom). The vertical lines divide the values for q_0. (Left) PG Map (Right) VM Map

are the best in most situations, but there are some others where the best value for q_0 is 0.1: those where the safety has not the highest priority (F_sFast and F_sMean), and the CSTR method is used. It is important to remark that both maps have small safe zones, so it is difficult for solutions move inside them if the security is not prioritized. In addition, the CSTR rule promote the exploitation and the value q_0 also controls it. A high value for this parameter boost again the exploitation, so it is difficult to find the safe zone in the map and so, the cost in energy (which depends on safety) is high. So the best solutions in this objective, when using the CSTR method, are reached for low values of q_0 (0.1), which promote the exploration. On the other hand, the DSTR method explores in a wider range of zones (solutions), so the combination with a high value for q_0 yields good results for all the sources.

On the other hand, it is reasonable to consider as many number of iterations (N_{Its}) as possible, since the solution can only improve, but the most delicate is the choice of the best value for the number of ants (N_{Ants}), since it implies a higher exploration component, so high values works very well when there is also a higher exploitation factor (high q_0). But in the previously commented exceptional cases, which yield their best solutions for low values of q_0, a low number of ants is suggested, in order to avoid a very high exploration component. It means that the parameters q_0 and N_{Ants} are correlated and, in the CSTR method (where there is an extra exploitation factor), they should take low values for the sources with less priority and high values otherwise.

It is important to notice that these results also depend on the map, because its associated 'difficulty' (the number of possible safe and fast solutions) could determine whether a higher exploitation or exploration component are needed. In addition, these results also depend on the objectives with a higher priority (they are be better for these objectives), so we should take into account the results of the Mean source with a higher level of relevance. The last factor to consider is the influence of λ parameter in the results, because this value drives the search to a determinate zone of the solutions space (related to the objective with higher priority) and yielding worse results for the other objective. In this study we use λ=0.5, to set the same relevance to both objectives.

Finally, as a summary:

- CSTR method: the best values for the parameters would be: q_0**=0.8,** N_{Its}**=2200,** N_{Ants}**=80** for the objectives with higher priority ($F_f Fast$ and $F_s Safe$), and, in these maps (which apparently have more possible fast paths than safe paths), for $F_f Mean$. The best for the safe objective (when it does not have the highest priority) would be: q_0**=0.1,** N_{Its}**=—,** N_{Ants}**=—**, using a high value in iterations and low in ants, or the other way around.
- DSTR method: the best values would always be: q_0**=0.8,** N_{Its}**=2200,** N_{Ants}**=80**, and a smaller number of ants if safety gets a lower priority ($F_f Safe$ and $F_s Mean$).

As a clarification, these values would yield good performance in most maps (if the characteristics and size are close to these ones), but they should be chosen according to the difficulty of the map, making a prior study to determine if it is globally a 'safe' or a 'fast' map.

VI. Conclusions and Future Work

In this work, we have made an statistical analysis of the influence of the parameters in the performance of a MOACO algorithm (named hCHAC), which finds the fastest and safest path (with relative importance set by the user) for a simulated military unit in a realistic battlefield (real terrains, enemies, visibility constraints, weapons impact zones). We have considered two different state transition rules (CSTR and DSTR), which is equivalent to two different approaches, and we have run the experiments in two different maps. We have applied the 1-Factor ANOVA statistical method, and we have completed the analysis using an analysis of mean (also applying t-Student test when needed) for all the experiments.

Three parameters have been studied: q_0, proportion factor of ACS, which determines the balance between exploration and exploitation in the search; the *Number of Iterations* (N_{Its}), which usually promotes the exploitation (the solutions can be improved more times); and finally, the *Number of Ants* (N_{Ants}), which usually promotes the exploration (they can explore more areas of the space of solutions).

We have reached some interesting conclusions and some relations between them have been found (q_0 is correlated with N_{Ants} in some cases when using the CSTR). In addition, we have identified the best set of values for all of them in these maps and for the two methods, but we also have discovered that some of these values depends on the objective with higher priority and on the features and 'difficulty' of the map (the number of possible safe and fast solutions).

Future work will study general parameters such as α, β and ρ, as well as other parameters particular to our algorithm. We are also working on a method to automatically compute the difficulty of a map, so that parameter settings can be adjusted to it.

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