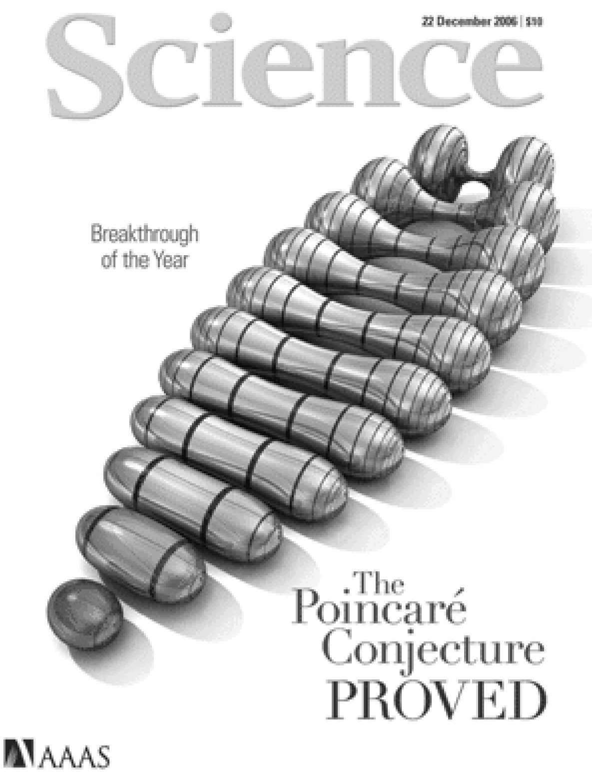
Demostración de Hamilton-Perelman de la Conjetura de Poincaré



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Málaga, 16 de abril de 2007

LENGUAJES Y CIENCIAS DE LA COMPUTACIÓN
UNIVERSIDAD DE MÁLAGA

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Serie de conferencias

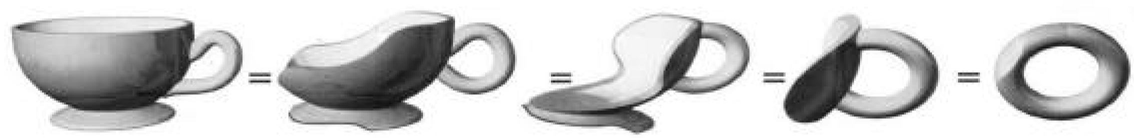
1. **Demostración de la conjetura**
    - Lunes 16 de abril (10:30)
2. **Flujo de Ricci-Hamilton**
    - Viernes 20 de abril (10:00)
3. **Solitones de Ricci y singularidades**
    - Lunes 23 de abril (10:30)
4. **Aportaciones de Perelman**
    - Viernes 27 de abril (10:00)

Illustration by Robert Neubecker.

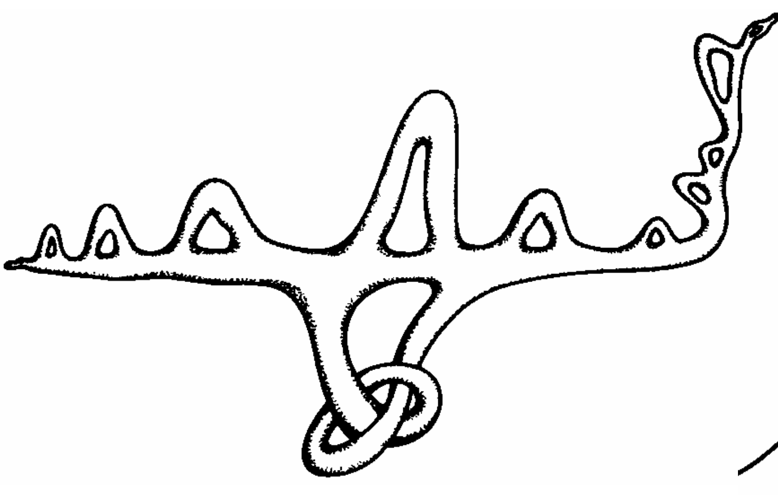
Demostración de la conjetura

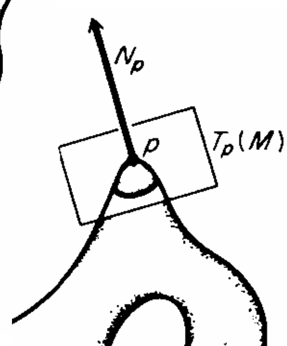
- Topología, geometría y análisis geométrico
- ¿Qué es la conjetura de Poincaré?
- La historia en el s. XX de la conjetura
- El programa de Hamilton-Yau
- Ideas sobre la demostración
  de Grigori Perelman
- Breve historia de la demostración

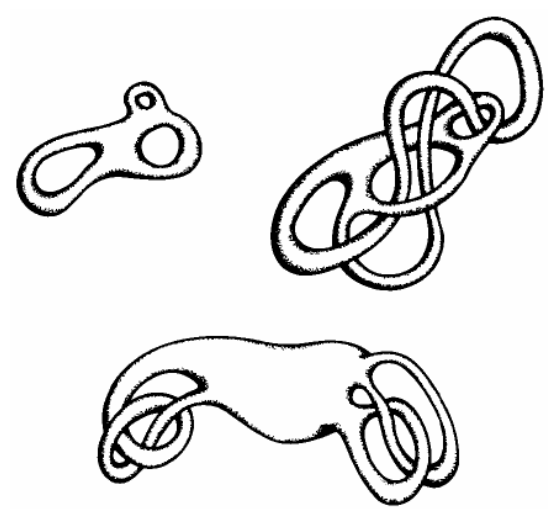
Topología



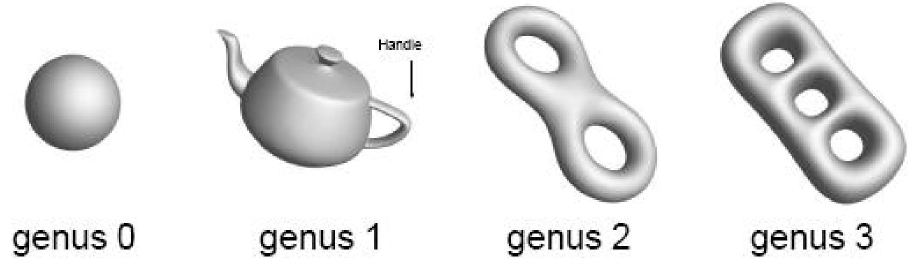
- Variedad topológica
  - (hiper)superficie localmente euclídea
  - Todo punto tiene (hiper)plano tangente.
- ¿Cuándo dos variedades son homeomorfas (equivalentes)?
  - Existe transformación continua con inversa continua entre ellas





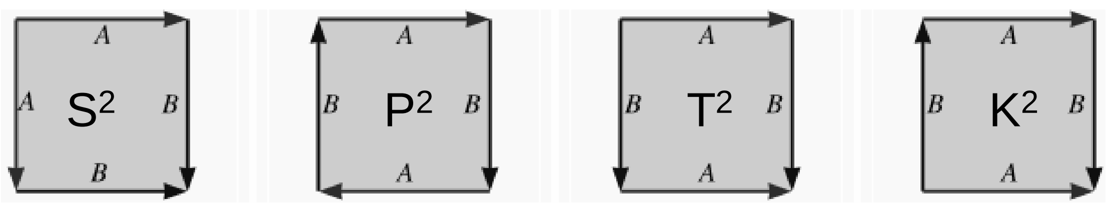


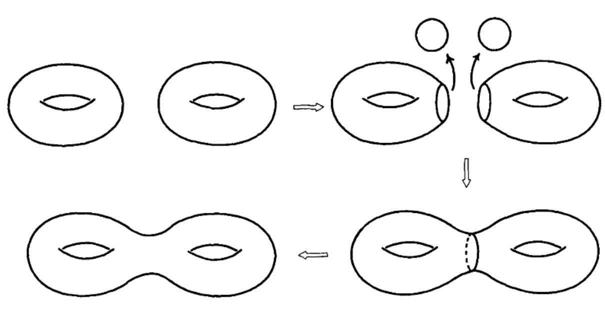
Topología



- Clasificación de todas las superficies (variedades 2D)
    - Gran logro de la matemática del siglo XIX
    - Sólo hay que contar el número de agujeros (número de Betti)
- Variedades compactas (cerradas y acotadas, sin borde) y orientables (con dos caras), no consideramos

Banda Möbius o la botella de Klein





1860's

suma conexa

S^2

T^2

$T^2 \# T^2$

$T^2 \# T^2 \# T^2$

$T^2 \# T^2 \# T^2 \# T^2$

P^2

$P^2 \# P^2$

$P^2 \# P^2 \# P^2$

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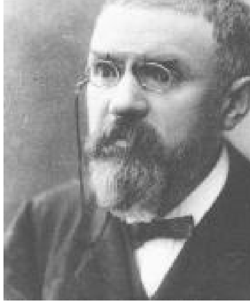
Conjetura de Poincaré



- Cuestión fundamental de la topología (de variedades)

  Dada una variedad n-dimensional cualquiera, ¿es homeomorfa a la esfera? ¿Cómo puedo saberlo?

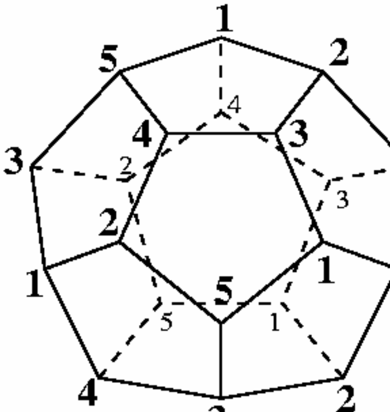
- 1900 – "teorema erróneo" de Poincaré

Tout polyèdre qui a tous ses nombres de Betti égaux à **1** *et tous ses tableaux T_q bilatères est simplement connexe, c'est-à-dire homéomorphe à l'hypersphère.*

If a closed 3-dimensional manifold has the homology of the sphere S^3, then it is necessarily homeomorphic to S^3.

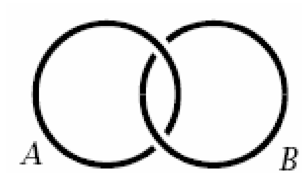
- 1904 – encuentra un contraejemplo:

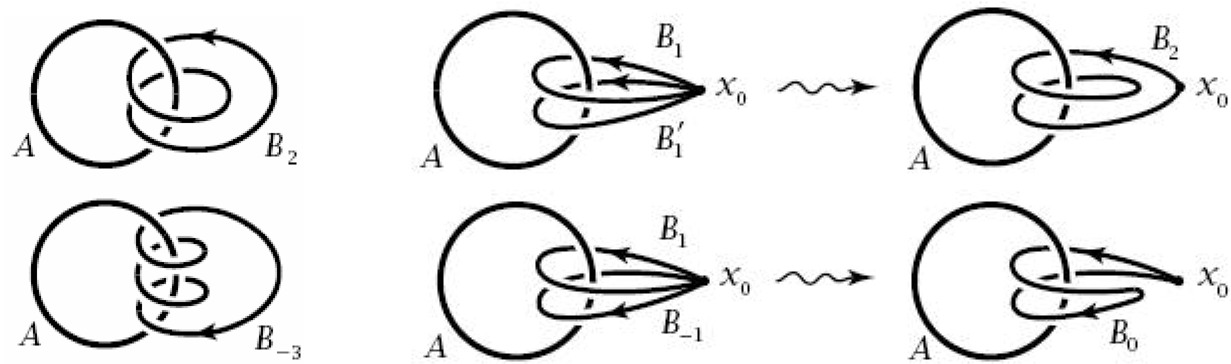
  "esfera homológica de Poincaré"



Conjetura de Poincaré

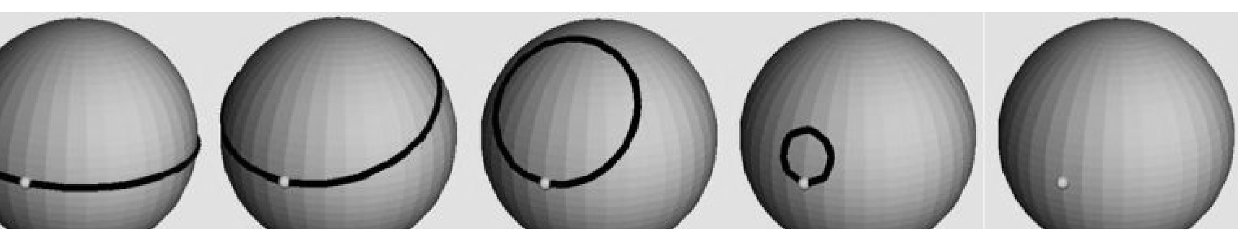
- Homotopía: grupo fundamental

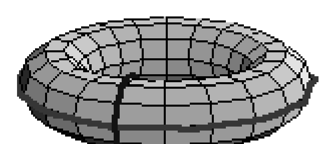




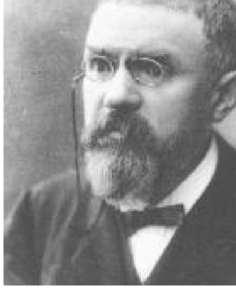
Grupo Abeliano

  - Grupo fundamental (finito) y trivial : Esfera
  - Variedad simplemente conexa





Conjetura de Poincaré



- La esfera homológica de Poincaré no es simplemente conexa (tiene grupo fundamental finito pero no trivial):

  Conjetura: es el único ejemplo que existe

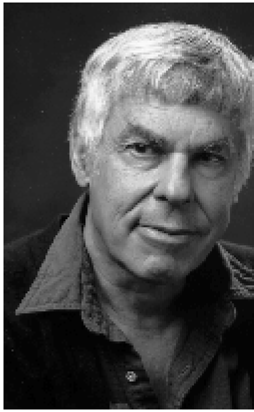
- Formulación correcta de la conjetura 1904

Il resterait une question à traiter :

Est-il possible que le groupe fondamental de V se réduise à la substitution identique, et que pourtant V ne soit pas simplement connexe?

If a closed 3-dimensional manifold has trivial fundamental group, must it be homeomorphic to the 3-sphere?

CP en dimensión alta



- Stephen Smale (ca. 1960) la demostró para n>4
  (primero en n=7, luego n=6, 5, y finalmente n>=5)
  Medalla Fields 1966
  (Wallace, Zeeman, Stallings)
- Idea: "desanudar nudos" en n>=5 en variedades simplemente conexas

- Siempre posible con un pequeña perturbación gracias a las dimensiones adicionales. Muy "duro" en n=5. No funciona en n<5, no hay "dimensiones" suficientes.

CP en dimensión alta

- Michael Freedman (1982) la demostró para n=4

  Medalla Fields 1986

- Idea: aprovechar que en n=4, las variedades

  $$\text{DIFF} = \text{PL} \neq \text{TOP}$$

- Utilizó desarrollos previos para el invariante de Kirby-Siebenmann que caracteriza las variedades TOP que no están en PL
- Es una demostración muy técnica y difícil de explicar

- Desafortunadamente, para n=3, DIFF = PL = TOP

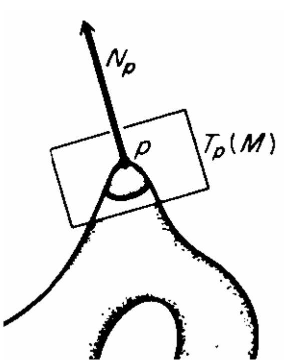
Geometrizar una variedad

- Espacio euclídeo: longitudes y ángulos medidos mediante un producto escalar

$$\mathbf{x}\cdot\mathbf{y} = \sum_{i=1}^{n} x_i y_i = x_1 y_1 + x_2 y_2 + \cdots + x_n y_n. \qquad \|\mathbf{x}\| = \sqrt{\mathbf{x}\cdot\mathbf{x}} = \sqrt{\sum_{i=1}^{n} (x_i)^2}. \qquad \theta = \cos^{-1}\left(\frac{\mathbf{x}\cdot\mathbf{y}}{\|\mathbf{x}\|\,\|\mathbf{y}\|}\right)$$

- Un producto escalar general : B es matriz simétrica

$$\mathbf{u}^T \mathbf{B}\mathbf{u} = \sum_{i,j=1}^{n} B_{ij} u^i u^j$$



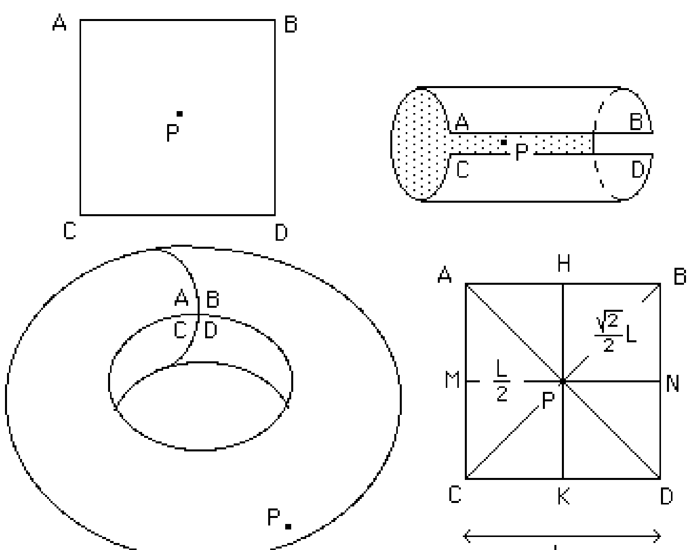
Definition 1.1. Let M be an n-dimensional manifold. A *Riemannian metric* g on M is a smooth section of $T^*M \otimes T^*M$ defining a positive definite symmetric bilinear form on T_pM for each $p \in M$. In local coordinates $(x^1, \cdots, x^n)$, one has a natural local basis $\{\partial_1, \cdots, \partial_n\}$ for TM, where $\partial_i = \frac{\partial}{\partial x^i}$. The metric tensor $g = g_{ij} dx^i \otimes dx^j$ is represented by a smooth matrix-valued function

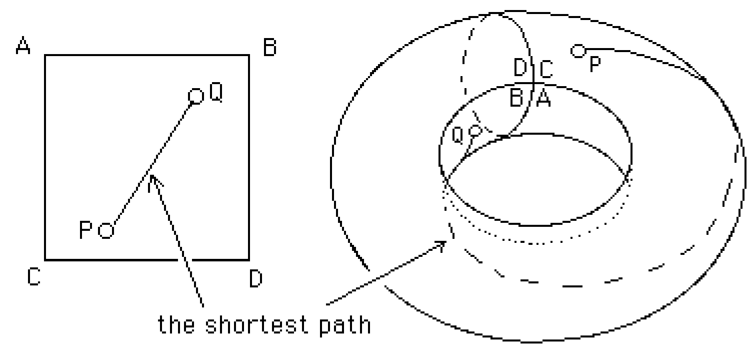
$$g_{ij} = g(\partial_i, \partial_j).$$

The pair (M, g) is a *Riemannian manifold*. We denote by (g^{ij}) the inverse of the matrix (g_{ij}).

Geometrizar una variedad

- Toda variedad topológica admite infinitas métricas suaves, aunque tiene asociada una métrica "natural"
- Conceptos "intrínsecos" como la longitud de una curva, un área, un volumen, … la curvatura intrínseca, … dependen de la métrica.
- Toro 2D plano vs. Toro 2D sumergido en 3D





Existe métrica homogénea para el toro con curvatura ¡¡ nula !! ¿ intuitivo ?

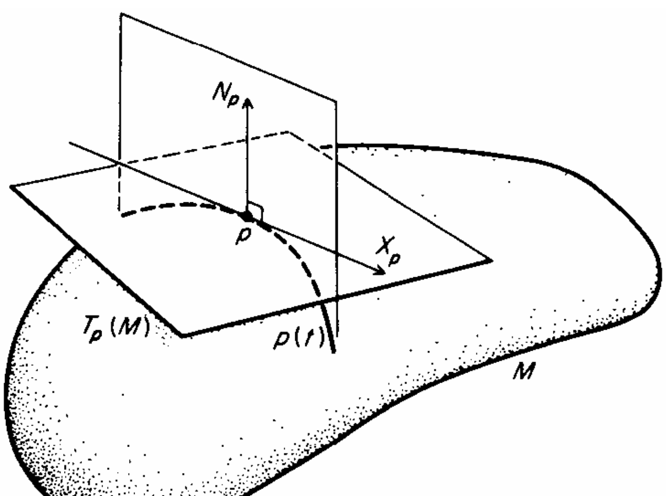
Curvatura de Riemann

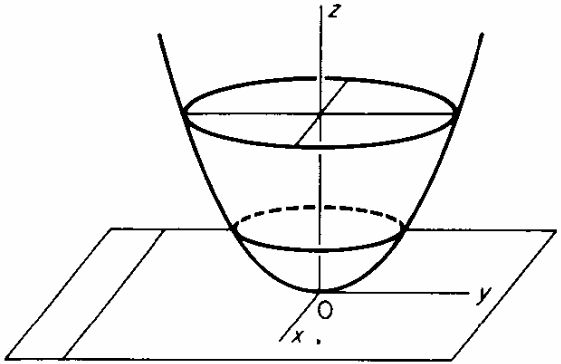
- Curvatura gaussiana para variedades en n=2

Let $f(x, y)$ be expanded in Taylor series at $(0, 0)$. Then

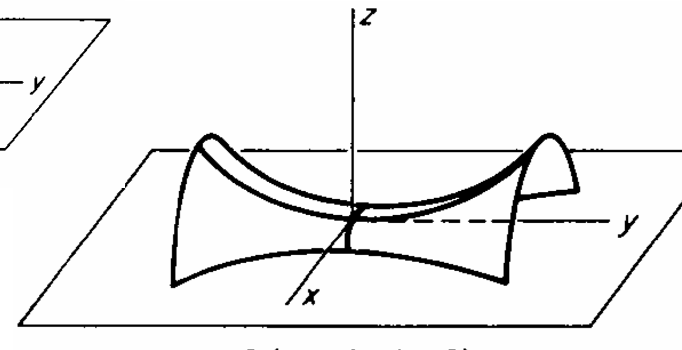
$$z = f(x, y) = f_{xx}(0, 0)x^2 + f_{yy}(0, 0)y^2 + R_2 ,$$

where R_2 contains terms of higher order. Let $f_{xx}(0, 0) = a$ and $f_{yy}(0, 0) = b$. Then we see that the normal sections of $z = ax^2 + by^2$ have the same sectional curvatures at p as does the given surface. Therefore the quadric surfaces must give typical examples.





$ab > 0$ $(a > 0,\ b > 0)$



Curvatura de Riemann

- Tensor de curvatura de Riemann en coordenadas locales

$$\mathcal{R}(\partial_i, \partial_j, \partial_k, \partial_l) = R_{ijkl} \qquad g_{ij}(x) \;=\; \delta_{ij} + \frac{1}{3} R_{iklj} x^k x^l +$$

- La "interpretación" geométrica de este tensor "más sencilla" requiere el concepto de curvaturas seccionales (curvaturas a la Gauss de planos 2D en el hiperplano tangente a M)

The *sectional curvature* of a 2-plane $P \subset T_pM$

$$K(P) = \mathcal{R}(X, Y, X, Y),$$

- El tensor de curvatura "intrínseca" de Riemann es poco intuitivo porque es invariante a difeomorfismos:
  - Una curva tiene curvatura intrínseca ¡¡ nula !!

Curvatura de Riemann

- Tensor de curvatura de Ricci

$$Ric(X,Y) = Ric_g(X,Y) = g^{kl} R(X, \partial_k, Y, \partial_l).$$

$$Ric = Ric_{ij} dx^i \otimes dx^j, \quad Ric_{ij} = Ric(\partial_i, \partial_j).$$

- Escalar de curvatura

$$R = R_g = \mathrm{tr}_g Ric = g^{ij} Ric_{ij}.$$

- R determina Ric y Riemann en n=2
- Ric determina Riemann en n=3
- Riemann es necesario sólo en n>3

Geometrización de variedades

- Teorema de homogeneización (s. XIX): Toda variedad topológica admite una métrica (geometría riemanniana) homogénea tal que tiene curvatura constante
  - Positiva igual a 1 => difeomorfa a la esfera
  - Nula (igual a 0) => difeomorfa al plano euclídeo
  - Negativa igual a -1 => difeomorfa al plano proyectivo

Theorem 1 *If X^n is a simply connected, complete Riemannian manifold with constant sectional curvature* $+1$, 0 *or* -1 *then* X *is isometric to* S^n, $\mathbb{E}^n$ *or* $\mathbb{H}^n$ *respectively.*

In particular the only homogenous, simply connected Riemannian surfaces are S^2, $\mathbb{E}^2$ and $\mathbb{H}^2$; these all have compact quotients, so they are the three 2-dimensional geometries.

Conjetura de Thurston

Decomposition of 3-manifolds:

Assume that M is closed (compact, no boundary).

Step 1: Connected sum decomposition of M into *prime* pieces (closed manifolds which cannot be decomposed further).

Step 2. If M is prime, consider a *toral* decomposition of M along *incompressible* [1] tori into *simple* pieces (the ones which cannot be decomposed further). Note that simple pieces typically have nonempty toral boundary.

Both decomposition processes terminate (Kneser, Haken: theory of normal surfaces).

Uniqueness of the decompositions: (1) Components of the connected sum decomposition are uniquely determined by M (Milnor). (2) The toral decomposition is unique up to isotopy if we consolidate simple pieces into maximal geometric pieces (Jaco, Shalen; Johannson).

Similar decompositions exist for compact manifolds with boundary.

Thurston's Geometrization Conjecture (GC): *Each prime closed 3-manifold M is either geometric or its simple pieces are geometric.*

3-dimensional geometries

- $S^3, \mathbb{E}^3, \mathbb{H}^3$, are the constant (sectional) curvature geometries.
- $S^2 \times \mathbb{R}, \mathbb{H}^2 \times \mathbb{R}$ are the product geometries.
- $Nil, Sol, \widetilde{SL_2}(\mathbb{R})$ are the twisted product geometries.

Conjetura de Thurston

- Michael Thurston (Medalla Fields 1986)

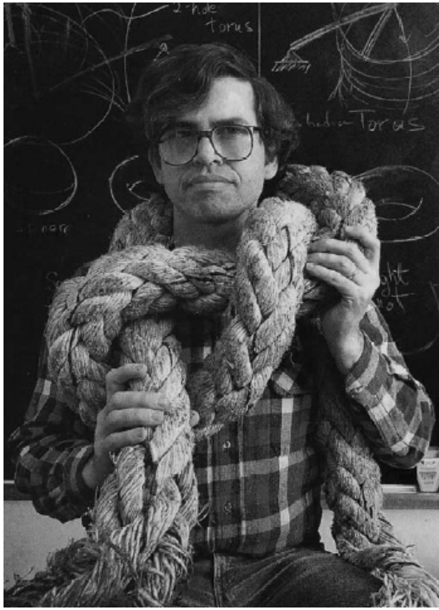
4. The Thurston Geometrization Conjecture

In the two-dimensional case, each smooth compact surface can be given a beautiful geometrical structure, as a round sphere in the genus zero case, as a flat torus in the genus 1 case, and as a surface of constant negative curvature when the genus is 2 or more. A far-reaching conjecture by William Thurston in 1983 claims that something similar is true in dimension 3 [46].

There are eight possible three-dimensional geometries in Thurston's program. Six of these are now well understood,[5] and there has been a great deal of progress with the geometry of constant negative curvature.[6] The eighth geometry, however, corresponding to constant positive curvature, remains largely untouched. For this geometry, we have the following extension of the Poincaré Conjecture.

Thurston Elliptization Conjecture. *Every closed 3-manifold with finite fundamental group has a metric of constant positive curvature and hence is homeomorphic to a quotient* S^3/Γ, *where* $\Gamma \subset \mathrm{SO}(4)$ *is a finite group of rotations that acts freely on* S^3.

The Poincaré Conjecture corresponds to the special case where the group $\Gamma \cong \pi_1(M^3)$ is trivial. The possible subgroups $\Gamma \subset \mathrm{SO}(4)$ were classified long ago by [19] (compare [23]), but this conjecture remains wide open.

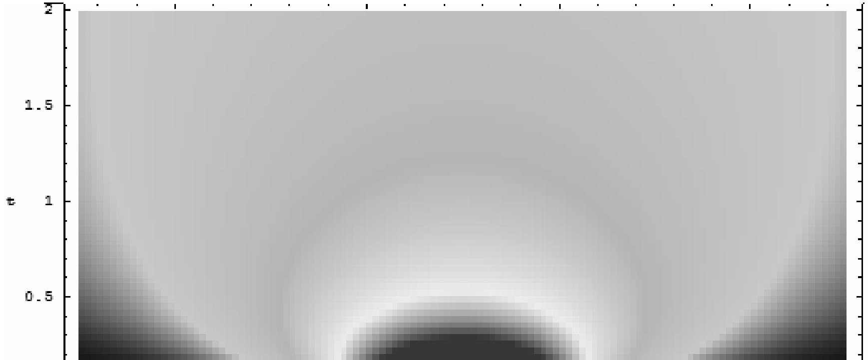


GEOMETRY and the IMAGINATION

A conference in honor of Bill Thurston's 60th birthday

Programa de Hamilton-Yau

- Richard Hamilton
  - Joven postdoc, "creador" del flujo de Ricci para demostrar la conjetura de eliptización de Thurston (entonces muy de moda).
- Shing-Tung Yau
  - Medalla Fields en 1982 por su demostración de la conjetura de Calabi, colaboró con Hamilton en cómo atacar la conjetura.
- Idea: Queremos homogeneizar la curvatura. La ecuación del calor homogeneiza la temperatura, ¿por qué no usar una ecuación del calor (parabólica) para la curvatura?



Programa de Hamilton-Yau

- Ecuación del flujo de Ricci (el volumen decrece)

$$\frac{\partial g}{\partial t} = -2\,\mathrm{Ric}(g)$$

$$R_{ij} = -\frac{1}{2}\Delta g_{ij} + \text{lower order terms}$$

(operador Laplace-Beltrami)

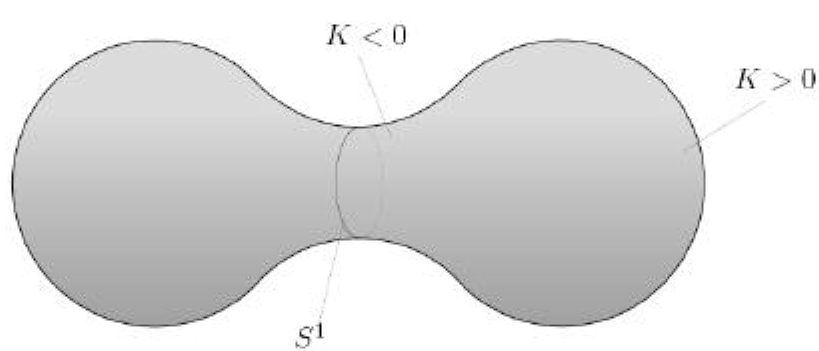
- Ecuación del flujo de Ricci normalizado (volumen const.)

$$\frac{\partial g}{\partial t} = -\mathrm{Ric} + \frac{R}{2}g$$

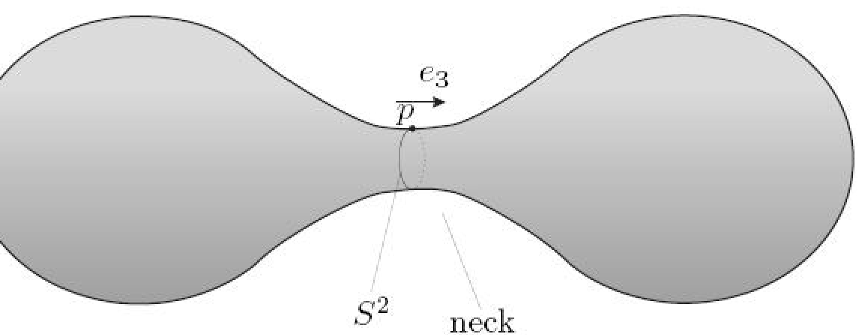
Flujo de Ricci

- Hamilton (1982): Teorema de homogeneización en 2D

Gauss curvature K as $\mathrm{Ric}(g) = Kg$.



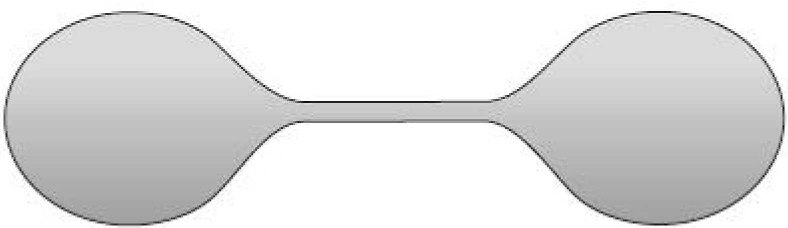
- En 3D es más difícil porque se producen singularidades

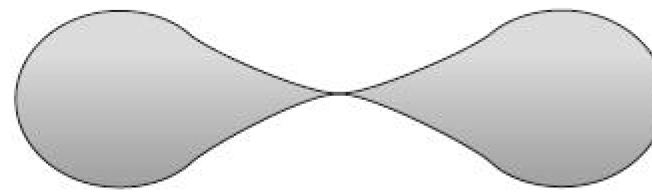


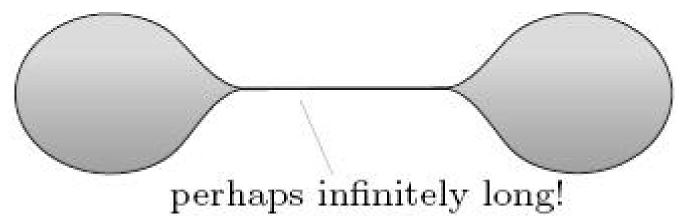
$$\mathrm{Ric}(e_1, e_1) = K_{e_1 \wedge e_2} + K_{e_1 \wedge e_3} = \text{very positive}$$
$$\mathrm{Ric}(e_2, e_2) = K_{e_2 \wedge e_1} + K_{e_2 \wedge e_3} = \text{very positive}$$
$$\mathrm{Ric}(e_3, e_3) = K_{e_3 \wedge e_1} + K_{e_3 \wedge e_2} = \text{slightly negati}$$

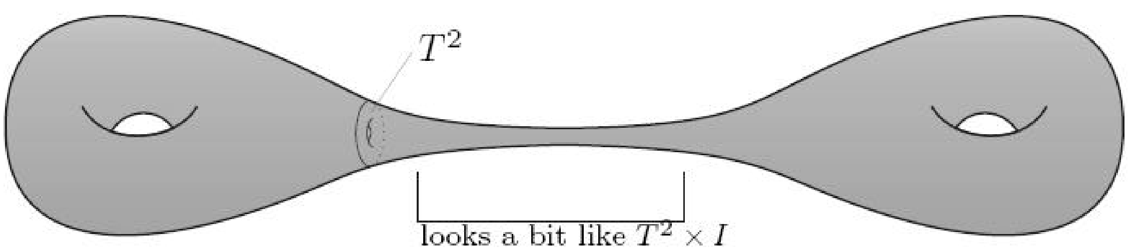
Flujo de Ricci

- Teorema de Pinzamiento de Hamilton-Ivey: la curvatura positiva tiende a aumentar reduciendo el volumen, y la negativa tiende a hacerse más negativa, pero menos rápido que la positiva: 3 posibilidades, a priori,









Flujo de Ricci

- Existencia y unicidad de la solución (Hamilton, 1982)
  - Iteración de Nash-Moser
  - DeTurck, transformación gauge a una ecuación del calor
  - Resultados LOCALES
- Hamilton (1982) caso particular de la conj. Eliptización
  - Si la variedad tiene curvatura seccional mínima positiva, entonces el flujo existe para todo tiempo y la métrica se homogeneiza.
  - Demostración basada en un principio del máximo para tensores
- Problema: si hay curvatura negativa, hay blow-up
  - La curvatura tiende a infinito en ciertos puntos

Flujo de Ricci con cirugía

- Clasificación de todas las superficies (variedades 2D)
- ¿Cuándo dos variedades son homeomorfas (equivalentes)?
  - Existe transformación continua con inversa continua entre ellas

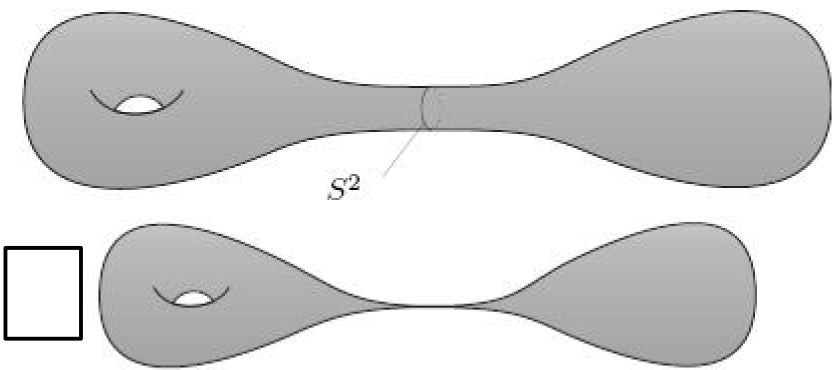


Figure 1.6: Neck pinch

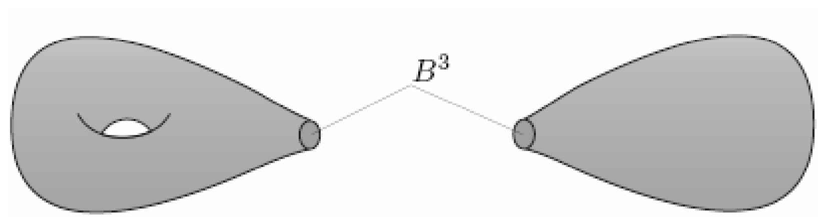


Figure 1.7: Surgery

  - Por supuesto, he de hacer "bien" la cirugía

Flujo de Ricci con cirugía

Problemas encontrados por Hamilton

1. Las singularidades, ¿todas se producen al mismo T*? Si no, no nos daría "tiempo" para hacer la cirugía
2. ¿El número de singularidades de M en T* es finito?
3. Hay que clasificar todos los tipos de singularidades, e indicar cómo aplicar la cirugía a cada tipo
4. Los tiempos de blow-up T* no pueden acumularse, es decir, en un intervalo finito sólo habrá un número finito

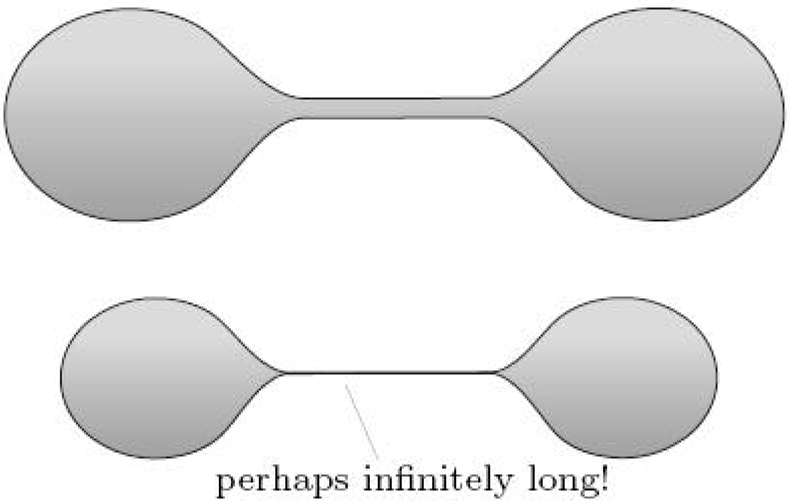
Hamilton realizó casi todo el programa "módulo" ciertas conjeturas "razonables" que no fue capaz de demostrar

Singularidades del flujo de Ricci

1. Demostrado por Hamilton. Típico en ecuaciones parabólicas no lineales
2. Requiere una acotación del llamado radio de inyectividad. Requiere una acotación coercitiva de la curvatura. Fue obtenida por Perelman, demostrando que "sólo caben" un número finito de singularidades.
3. Clasificación de las singularidades: son solitones del flujo de Ricci (estacionarios en T*) y tipo gradiente un "poquito" antes (soluciones ancianas). Hamilton encontró demasiadas "posibles" singularidades, entre ellas el solitón cigarro.

Singularidades del flujo de Ricci

- El solitón cigarro: la pesadilla de Hamilton
  - Conjeturó que no pueden darse, pero no lo pudo demostrar



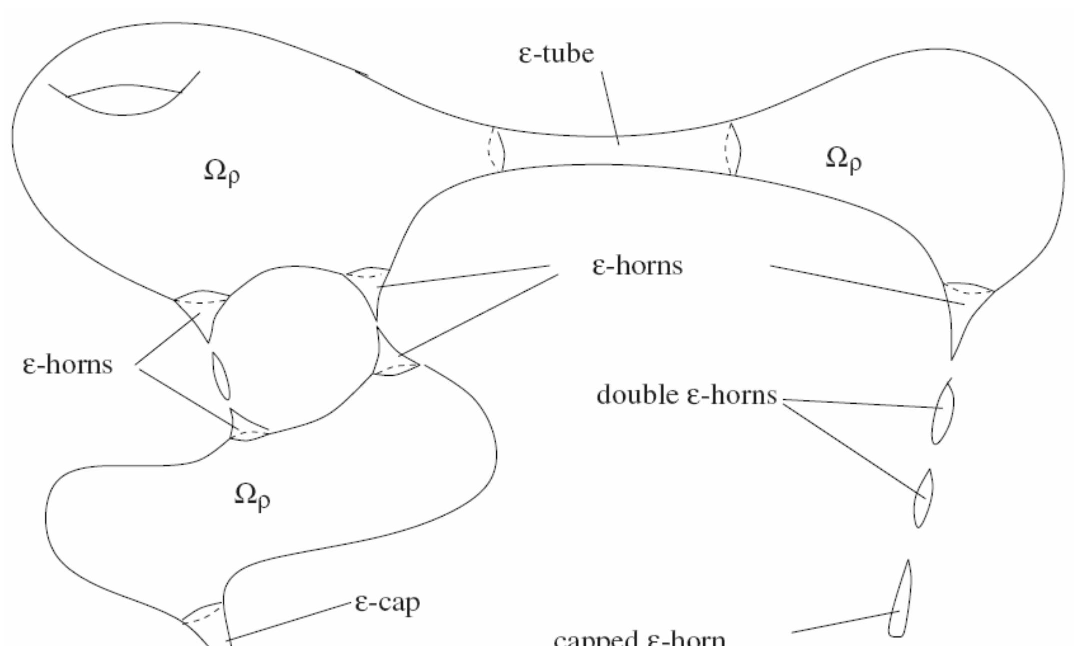
  - "Cilindro" que se contrae uniformemente generando infinitas singularidades. La cirugía no funciona con estas soluciones

Entropía de Perelman

- Demostró que el flujo de Ricci "es" un flujo gradiente
  - Entropía W
  - Volumen reducido
  - Magnitudes monótonas bajo flujo de Ricci
- El solitón cigarro ¡¡ no puede darse !! Viola la entropía

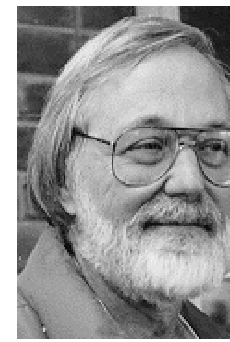
- Clasificación de singularidades

Teorema de no colapso => cirugía precisa y fina



Yo y la conjetura de Poincaré

- 1992 "Topología Algebraica" Aniceto Murillo (UMA)
  - La demostración está próxima (sin detalles)
- 2000 "Premios del milenio"
  - 1 millón de dólares para los "mejores"

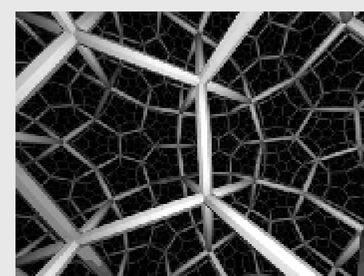


Millennium Problems

In order to celebrate mathematics in the new millennium, The Clay Mathematics Institute of Cambridge, Massachusetts (CMI) has named seven *Prize Problems*.

- Birch and Swinnerton-Dyer Conjecture
- Hodge Conjecture
- Navier-Stokes Equations
- P vs NP
- Poincaré Conjecture
- Riemann Hypothesis
- Yang-Mills Theory

- Official Problem Description – John Milnor
- Lecture by Cameron Gordon University of Texas (video)



"Demostraciones" en 2002 conjetura de Poincaré

Dunwoody, M. J. "A Proof of the Poincaré Conjecture." http://www.maths.soton.ac.uk/pure/viewabstract.phtml?entry=655. Rev. Apr. 9, 2002.

Nikitin, S. "Proof of the Poincare Conjecture" 22 Oct 2002. http://arxiv.org/abs/math.GT/0210334/.

Perelman, G. "The Entropy Formula for the Ricci Flow and Its Geometric Application" 11 Nov 2002. http://arxiv.org/abs/math.DG/0211159/.

thors: **Grisha Perelman**
omments: 39 pages
bj-class: Differential Geometry
SC-class: 53C

*St.Petersburg branch of Steklov Mathematical Institute, Fontanka 27, St.Petersburg 191011, Russia. Email: perelman@pdmi.ras.ru or perelman@math.sunysb.edu ; I was partially supported by personal savings accumulated during my visits to the Courant Institute in the Fall of 1992, to the SUNY at Stony Brook in the Spring of 1993, and to the UC at Berkeley as a Miller Fellow in 1993-95. I'd like to thank everyone who worked to make those opportunities available to me.

We present a monotonic expression for the Ricci flow, valid in all dimensions and without curvature assumptions. It is interpreted as an entrop
for a certain canonical ensemble. Several geometric applications are given. In particular, (1) Ricci flow, considered on the space of riemannia
metrics modulo diffeomorphism and scaling, has no nontrivial periodic orbits (that is, other than fixed points); (2) In a region, where singularity
forming in finite time, the injectivity radius is controlled by the curvature; (3) Ricci flow can not quickly turn an almost euclidean region into a v
curved one, no matter what happens far away. We also verify several assertions related to Richard Hamilton's program for the proof of
Thurston geometrization conjecture for closed three-manifolds, and give a sketch of an eclectic proof of this conjecture, making use of earlie
results on collapsing with local lower curvature bound.

"Demostraciones" en 2002 conjetura de Poincaré

A Proof of the Poincaré Conjecture ?
by
M.J.Dunwoody

Dunwoody, M. J. "A P
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"Demostraciones" en 2002 conjetura de Poincaré

A Proof of the Poincaré Conjecture ?
by
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Dunwoody, M. J. "A P ... http://www.maths.so ... ry=655. Rev. Apr. 9, 2002.

The Poincaré conjecture for stellar manifolds

Sergey Nikitin

Nikitin, S. "Proof of the Poi ... 2002. http://arxiv.org/abs/math

Perelman, G. "The Entropy Formula for the Ricci Flow and Its Geometric Application" 11 Nov 2002. http://arxiv.org/abs/math.DG/0211159/.

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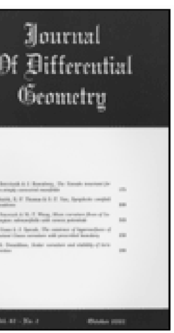
"Demostraciones" en 2002 conjetura de Poincaré

- ¿Quién "era" ese Grisha Perelman?
- Grigori Perelman: alumno de Alexandrov
- Considerado un genio en geometría riemanniana
- "Famoso" por demostrar la conjetura (soul conjecture) de Cheeger-Gromoll (1972) en 1994



  - If M is a complete noncompact Riemannian manifold with nonnegative sectional curvature, and there is one point where all of the sectional curvatures are positive, then M is diffeomorphic to Euclidean space.

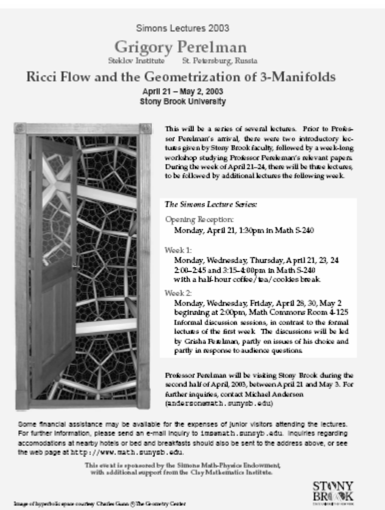
G. Perelman
Proof of the Soul Conjecture of Cheeger and Gromoll
PDF file
p.209-212

- Premio EMS (European Mathematical Society) en 1996 => "rechazó" el premio

"Demostraciones" en 2002 conjetura de Poincaré

- ¿Es correcta la demostración?
- Extremadamente técnica
- Los expertos encontraron dificultades

"Que nos lo cuente" (gira americana en 2003)


Simons Lectures 2003

Grigory Perelman
Steklov Institute St. Petersburg, Russia

Ricci Flow and the Geometrization of 3-Manifolds

April 21 – May 2, 2003
Stony Brook University

This will be a series of several lectures. Prior to Professor Perelman's arrival, there were two introductory lectures given by Stony Brook faculty, followed by a week-long workshop studying Professor Perelman's relevant papers. During the week of April 21–24, there will be three lectures, to be followed by additional lectures the following week.

The Simons Lecture Series:

Opening Reception:
Monday, April 21, 1:30pm in Math S-240

Week 1:
Monday, Wednesday, Thursday, April 21, 23, 24
2:00–2:45 and 3:15–4:00pm in Math S-240
with a half-hour coffee/tea/cookies break.

Week 2:
Monday, Wednesday, Friday, April 28, 30, May 2
beginning at 2:00pm, Math Commons Room 4-125
Informal discussion sessions, in contrast to the formal lectures of the first week. The discussions will be led by Grisha Perelman, partly on issues of his choice and partly in response to audience questions.

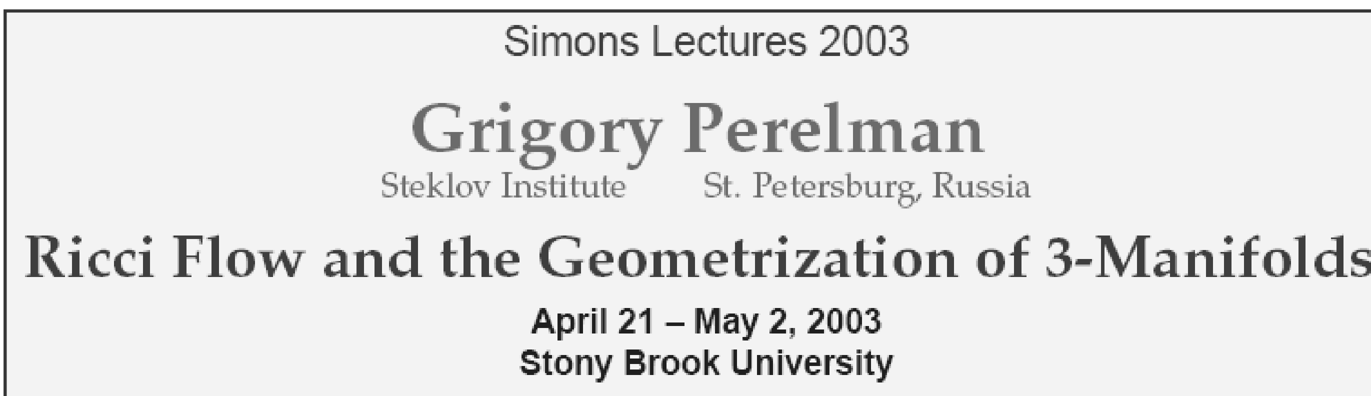
Professor Perelman will be visiting Stony Brook during the second half of April, 2003, between April 21 and May 3. For further inquiries, contact Michael Anderson (anderson@math.sunysb.edu)

Some financial assistance may be available for the expenses of junior visitors attending the lectures. For further information, please send an e-mail inquiry to ims@math.sunysb.edu. Inquiries regarding accomodations at nearby hotels or bed and breakfasts should also be sent to the address above, or see the web page at http://www.math.sunysb.edu.

This event is sponsored by the Simons Math-Physics Endowment, with additional support from the Clay Mathematics Institute.

STONY BROOK

Image of hyperbolic space courtesy Charles Gunn © The Geometry Center



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Perelman "continua" en 2003

Date: Mon, 10 Mar 2003 16:44:35 GMT (24kb)

Ricci flow with surgery on three-manifolds

Authors: **Grisha Perelman**
Comments: 22 pages
Subj-class: Differential Geometry
MSC-class: 53C

This is a technical paper, which is a continuation of math.DG/0211159.

Date: Thu, 17 Jul 2003 15:26:38 GMT (8kb)

Finite extinction time for the solutions to the Ricci flow on certain three-manifolds

Authors: **Grisha Perelman**
Comments: 7 pages
Subj-class: Differential Geometry
MSC-class: 53C

Date (v1): Sun, 10 Aug 2003 12:41:19 GMT (10kb)
Date (revised v2): Mon, 25 Aug 2003 15:03:39 GMT (9kb)
Date (revised v3): Tue, 15 Aug 2006 10:52:15 GMT (18kb)

Estimates for the extinction time for the Ricci flow on certain 3-manifolds and a question of Perelman

Authors: **Tobias H. Colding, William P. Minicozzi II**
Comments: published version
Subj-class: Analysis of PDEs
MSC-class: AP, DG, GT

Perelman "continua" en 2004

- Descripción del problema de John Milnor (2004)

http://www.claymath.org/millennium/Poincare_Conjecture/poincare.pdf

5. APPROACHES THROUGH DIFFERENTIAL GEOMETRY AND DIFFERENTIAL EQUATIONS[7]

In recent years there have been several attacks on the geometrization problem (and hence on the Poincaré Conjecture) based on a study of the geometry of the infinite dimensional space consisting of all Riemannian metrics on a given smooth three-dimensional manifold.

One approach by Michael Anderson, based on ideas of Hidehiko Yamabe [53], studies the *total scalar curvature* $\iiint_{M^3} R\,dV$ as a functional on the space of all smooth unit volume Riemannian metrics. The critical points of this functional are the metrics of constant curvature (see [1]).

A different approach, initiated by Richard Hamilton studies the *Ricci flow* [15, 16, 17], that is, the solutions to the differential equation

$$\frac{dg_{ij}}{dt} = -2R_{ij}\,.$$

If we start with a 3-manifold of positive Ricci curvature, Hamilton was able to carry out this program and construct a metric of constant curvature, thus solving a very special case of the Elliptization Conjecture. However, in the general case, there are very serious difficulties, since

Demostración en 2006

- Tres publicaciones oficiales de la demostración

ASIAN J. MATH.
Vol. 10, No. 2, pp. 165–492, June 2006

A COMPLETE PROOF OF THE POINCARÉ AND GEOMETRIZATION CONJECTURES – APPLICATION OF THE HAMILTON-PERELMAN THEORY OF THE RICCI FLOW*

HUAI-DONG CAO† AND XI-PING ZHU‡

Abstract. In this paper, we give a complete proof of the Poincaré and the geometrization conjectures. This work depends on the accumulative works of many geometric analysts in the past thirty years. This proof should be considered as the crowning achievement of the Hamilton-Perelman theory of Ricci flow.

Key words. Ricci flow, Ricci flow with surgery, Hamilton-Perelman theory, Poincaré Conjecture, geometrization of 3-manifolds

arXiv:math/0612069 [ps, pdf, other] :
Title: **Hamilton-Perelman's Proof of the Poinca**
Authors: **Huai-Dong Cao, Xi-Ping Zhu**
Comments: This is a revised version of the article
MSC-class: 53C21, 53C44

*Received December 12, 2005; accepted for publication April 16, 2006.

arXiv:math/0605667 [ps, pdf, other] :
Title: **Notes on Perelman's papers**
Authors: **Bruce Kleiner, John Lott**
Comments: 200 pages

arXiv:math/0610903 [ps, pdf, other] :
Title: **Perelman's proof of the Poincar\'e conjecture: a nonlinear PDE perspect**
Authors: **Terence Tao**
Comments: 42 pages, unpublished
MSC-class: 53C44

arXiv:math/0607607 [ps, pdf, other] :
Title: **Ricci Flow and the Poincare Conjecture**
Authors: **John W. Morgan, Gang Tian**
Comments: 493 pages with over 30 figures and 3 pages of front material
MSC-class: 53C44; 57M40; 57M50; 53C21

Lectures on the Ricci Flow
de Peter **Topping** - 2006 - 124 páginas
books.google.com - Acerca de este libro

Demostración en 2006

- Publicaciones oficiales de la demostración

ASIAN J. MATH.
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Lectures on the Ricci Flow
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Medalla Fields para Perelman

Sir John Ball

Grigori Perelman: *Por sus contribuciones a la geometría y su revolucionaria profundización en la estructura geométrica y analítica del flujo de Ricci.*

El nombre de Gregory Perelman se ha hecho familiar entre el público interesado en cuestiones científicas. Su trabajo del periodo 2002-2003 proporcionó una rompedora visión del estudio de las ecuaciones de evolución y sus singularidades. Y más significativo aún, sus resultados han proporcionado una forma de resolver dos importantes problemas de la topología: la conjetura de Poincaré y la conjetura de la geometrización de Thurston. En el verano de 2006, la comunidad matemática está aún en el proceso de comprobar su trabajo para asegurar que es completamente correcto y que ambas conjeturas pueden considerarse demostradas. Después de más de tres años de intensivo escrutinio, los mayores expertos no han encontrado serias objeciones al trabajo.

International Mathematical Union (IMU)

Fields Medal Prizewinners

2006

- Andrei Okounkov
- Grigori Perelman*
- Terence Tao
- Wendelin Werner

*Grigori Perelman declined to accept the Fields Medal.

Medalla Fields para Perelman

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The King of Spain with Tao, Werner, Okounkov and Kleinberg

Anuncio "oficial" de la demostración en Madrid 2006

e, 22 | **Laudatio**
:25 | **Speaker:** **John Lott,** *University of Michigan, Ann Arbor, USA*
:45 | The work of Grigori Perelman PDF

RICHARD HAMILTON

Richard Hamilton (Universidad de Columbia, Nueva York, EEUU) finalizó ayer su conferencia plenaria, la primera del ICM2006, diciendo que se sentía "increíble-

John Morgan (Columbia University, New York, USA, together with Gang Tian (Princeton University, USA), has written a book presenting a complete account of the proof of the Poincaré Conjecture based on Perelman's ideas.

La declaración de Hamilton al cierre de su charla podría entenderse como una confirmación de que finalmente la conjetura de Poincaré se ha demostrado. Hamilton es uno de los expertos no sólo que ha analizado el trabajo de Perleman, sino que ha desarrollado la herramienta que ha resultado central para el trabajo del matemático ruso –una técnica llamada 'flujo de Ricci'-.

Hamilton explicó que el trabajo de Perelman "es difícil de entender" y que el propio Perelman usa en algunos momentos el término "sketch". "Y un esbozo es una invitación a trabajar para completarlo, a buscar una manera de hacerlo mejore. Pero no hay ninguna voluntad de cri-

JOHN MORGAN: "IN 2003, PERELMAN SOLVED THE POINCARE CONJECTURE"

Escasos segundos después del comienzo de la conferencia, un aplauso sincero y unánime inundó la sala. John Morgan acababa de anunciar que Gregori Perelman había resuelto la conjetura de Poincaré. Esta

Anuncio "oficial" de la demostración en Madrid 2006

The experience I have had multiple times when reading Perelman is that I would read something and I wouldn't understand a word of it. Then I go home and think about it. If I didn't understand it I would talk to Tiang about it, to Hamilton... When I eventually understood it –hours later, days later, sometimes weeks later—I would ask myself, OK, if I had to explain its main points as a guide in one paragraph what would I do? So having had that experience over and over again and never finding that the paragraph that Perelman wrote deviated from an absolutely accurate if incredibly compressed description of the argument that I had understood to be completely correct, I conclude that Perelman had just decided for some reason to compress everything.

Is Perelman's proof complete?

I believe if you take all three papers by Perelman together they are 55 or 60 pages, and we wrote 473 pages. The first hundred pages were mostly background information and the rest was mostly unpacking and re-ordering what was contained in the 55 pages of Perelman.

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Have you met Perelman?

I met him when he made his tour to the United States in 2003, when he came to explain his ideas. I attended several of his lectures and then talked privately with him on several occasions, and after he went back to Russia I was in e-mail contact with him when I was struggling to understand his writings. And he was always very forthcoming and patient in explaining his ideas, so while he seems to be socially reclusive, in the mathematical context he was forthcoming and patient.

¿Recibirá Perelman el millón de dólares del premio del Milenio?

INTERNATIONAL CONGRESS OF **MATHEMATICIANS** MADRID **2006**

CARLSON: "PERELMAN CUMPLE LOS REQUISITOS PARA EL PREMIO DEL MILENIO"

¿recibirá Grisha Perelman, como muchos han pronosticado, uno de los 'premios del Milenio'? La respuesta podría tenerla Jim Carlson, presidente del Instituto Clay de Matemáticas. Algo que nadie podría responder, sin embargo, es si Pereleman aceptará el premio si efectivamente le es concedido.

¿Quiere decir eso que Perelman será el próximo Premio del Milenio, el galardón otorgado por el Instituto Clay para quien resolviese alguno de los denominados "Problemas del Milenio"?

Las reglas del Instituto dicen que hay que esperar dos años desde que se publique el trabajo y haya una aceptación general por parte de la comunidad matemática.

¿Cree usted que lo aceptará?

Eso no lo puedo decir: no tengo ni idea. Nosotros procederemos exactamente igual que el ICM cuando le ha premiado con la Medalla Fields. Creo que John Ball y el comité de las medallas Fields lo han hecho perfectamente. Ellos le otorgaron la medalla basándose en sus logros, sin pensar en la posible reacción de Perelman. Si el Instituto Clay decide ofrecerle el Premio del Milenio a Perelman, seguirá la misma filosofía.

El Instituto Clay (Massachusetts, EE.UU.) fue fundado en 1999 por el empresario bostoniano Landon T. Clay.