1 DYNASTICALLY OPTIMAL RECOMBINATION (DOR)

Let \( x \) and \( y \) be two individuals from a solution space \( S \). A recombination operator \( X \) can be defined as a function \( X: S \times S \times S \rightarrow [0, 1] \), where \( X(x, y, z) \) is the probability of generating \( z \) when recombining \( x \) and \( y \) using \( X \). Clearly,

\[
\forall x \in S, \forall y \in S : \sum_{z \in S} X(x, y, z) = 1 \quad (1)
\]

The Dynamic Potential of \( x \) and \( y \) is defined as

\[
\Gamma_{(x,y)} = \{ z | \forall \xi \in \Xi : z \in \xi \Rightarrow (x \in \xi) \land (y \in \xi) \} \quad (2)
\]

where \( \Xi \) is the set of basic formae.

A recombination operator is said to be transmitting iff \( \{z|X(x, y, z) > 0\} \subseteq \Gamma_{(x,y)} \). Now, let \( \phi : S \rightarrow R^+ \) be the target function (minimization is assumed). DOR is a transmitting recombination operator for which:

\[
\text{DOR}(x, y, z) > 0 \Rightarrow \forall w \in \Gamma_{(x,y)} : \phi(w) \geq \phi(z) \quad (3)
\]

Thus, no other solution in the dynastic potential is better than any solution generated by DOR. According to this definition, the use of DOR implies performing an exhaustive search in a small subset of the solution space. Such an exhaustive search can be efficiently done by means of a subordinate A*-like mechanism.

DOR uses optimistic estimations \( \hat{\phi}(\Psi) \) of the fitness of partially specified solutions \( \Psi \) (i.e., \( \forall z \in \Psi : \hat{\phi}(\Psi) \leq \phi(z) \)) for directing the search to promising regions. These solutions are incrementally constructed using the formae to which any of the parents belong. More precisely, let \( \Psi_0 = S \). Subsequently,

\[
\Psi_{t+1}^{2i} = \Psi_t^i \cap \Sigma(\Psi_t^i, x), \quad (4)
\]

\[
\Psi_{t+1}^{2i+1} = \Psi_t^i \cap \Sigma(\Psi_t^i, y) \quad (5)
\]

are considered. Whenever \( \bar{\phi} < \hat{\phi}(\Psi) \) (where \( \bar{\phi} \) is the fitness of the best-so-far solution generated during this process), the macro-forma \( \Psi \) is closed (i.e., discarded), hence pruning dynastically suboptimal solutions. Otherwise, the process is repeated for open macro-formae. Each \( \Sigma(\Psi, w) \) is termed a construction unit. These construction units are defined as

\[
\Sigma(\Psi, w) = \cap_{1 \leq i \leq g} \xi_j, w \in \xi_j, \quad (6)
\]

and their structure depends on the problem considered. The parameter \( g \) is called the granularity of the representation. It can be seen that the size of the set of solutions in which DOR searches is \( O(2^n/g) \), where \( n \) is the dimensionality of the representation.

The minimal value of \( g \) for a given representation is termed the basic granularity (e.g., \( g = 1 \) when the representation is orthogonal). If the computational complexity of DOR is too high for this basic granularity, \( g \) can be increased so as to make DOR combine larger portions of the ancestors.

Experimental results on the Brachystochrone design problem and the Rosenbrock function show a nearly-linear relation between the granularity of the representation and the reduction of the computational effort. Furthermore, it is shown that intermediate granularity values are better since low \( g \) is computationally prohibitive and high \( g \) reduces the chances for information interchange during recombination. This is verified on orthogonal and non-orthogonal separable representations exhibiting epistasis [2].

References