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Introduction

Many optical fibres are manufactured by melt-spinning processes. One of the two methods for the fabrication of high-bandwidth, low attenuation graded-index (GI) plastic optical fibres (POF) is called *co-extrusion*. In this process, monomers doped with low-molecular weight molecules and homogeneous monomers are polymerized as core and cladding or sheath polymers, respectively. The two materials are then melted in their respective parts and compounded in a die to fabricate a POF that has a concentric circular core/cladding structure. By heating the fiber in a diffusion section, a radial concentration profile of low-weight molecules is formed as a result of molecular diffusion. Solidification takes place between the plate and a rotating drum, and large extensions rates, rapid cooling, and high speeds are usually involved. Finally, the GI POF is obtained by winding it on a take-up reel.

A one-dimensional model of semi-crystalline, bicomponent fibres that includes molecular orientation and crystallization was developed by Ramos [1]. Such a model is governed by one-dimensional partial differential equations for the leading-order geometry, axial velocity component and temperature that were derived from the conservation equations of mass, linear momentum and energy under the assumptions that the fiber is slender ($\varepsilon=R_0/L \ll 1$) and the Reynolds and Biot numbers are small.

Formulation

We consider an axisymmetric, compound liquid jet such as the one shown schematically in Figure 1, consisting of two immiscible, incompressible (constant density) fluids which are assumed to be Newtonian.

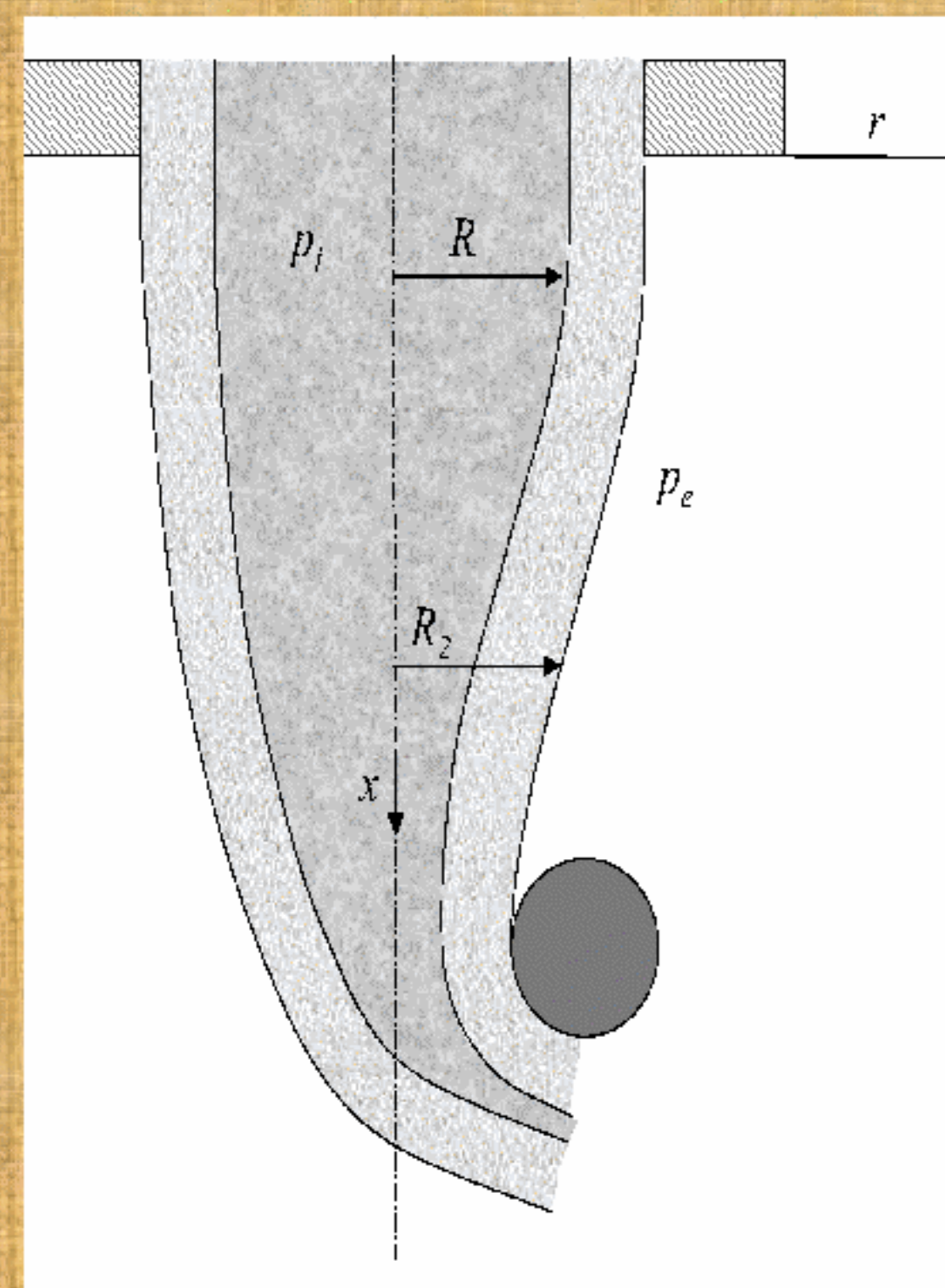


Figure 1. Schematic of compound fibre.

The fluid dynamics of bicomponent fibres are governed by the two-dimensional conservation equations of mass, linear momentum and energy,

$$\nabla \cdot \mathbf{v}_i = 0 \quad i = 1, 2, \quad (1)$$

$$\rho_i \left(\frac{\partial \mathbf{v}_i}{\partial t} + \mathbf{v}_i \cdot \nabla \mathbf{v}_i \right) = -\nabla p_i + \nabla \cdot \boldsymbol{\tau}_i + \rho_i \cdot \mathbf{F}_i^m \quad i = 1, 2, \quad (2)$$

$$\rho_i C_i \left(\frac{\partial T_i}{\partial t} + \mathbf{v}_i \cdot \nabla T_i \right) = k_i \Delta T_i + \boldsymbol{\tau}_i : \nabla \mathbf{v}_i \quad i = 1, 2, \quad (3)$$

plus appropriate boundary conditions at the nozzle exit ($x = 0$), downstream or take-up location ($x = L$), initial conditions, symmetry boundary conditions at $r = 0$, and kinematic, dynamic and thermal boundary conditions at the interfaces $r = R_1(t, x)$ and $r = R_2(t, x)$.

In the model presented here, we shall assume that the specific heat, C_i , and thermal conductivity, k_i , where $i = 1, 2$ denote the core and the cladding, respectively, are constant.

We model the solution of rod-like polymers as an ensemble of rigid dumbbells suspended in a Newtonian solvent viscosity. In such a solution, the deviatoric stress tensor is assumed to be

$$\boldsymbol{\tau}_p = \mu_{eff} (\nabla \mathbf{v} + (\nabla \mathbf{v})^T) + \boldsymbol{\tau}_p, \quad (4)$$

where \mathbf{v} is the velocity vector, μ_{eff} is an effective viscosity to be defined below, and

$$\boldsymbol{\tau}_p = 3ck_B T (-\lambda F(\mathbf{S})/\phi + 2\lambda((\nabla \mathbf{v})^T : \mathbf{S})(\mathbf{S} + \mathbf{I}/3)), \quad (5)$$

where \mathbf{I} is the identity or unit tensor, \mathbf{S} is the molecular orientation tensor, c is the number of polymeric units per unit volume, k_B is the Boltzmann constant,

$$\frac{\partial \mathbf{S}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{S} = (\nabla \mathbf{v})^T \cdot \mathbf{S} + \mathbf{S} \cdot \nabla \mathbf{v} + F(\mathbf{S}) + G(\nabla \mathbf{v}, \mathbf{S}), \quad (6)$$

which is traceless and diagonal tensor,

$$F(\mathbf{S}) = -\frac{\phi}{\lambda} ((1 - N/3)\mathbf{S} - N(\mathbf{S} \cdot \mathbf{S}) + N(\mathbf{S} : \mathbf{S})(\mathbf{S} + \mathbf{I}/3)), \quad (7)$$

$$G(\nabla \mathbf{v}, \mathbf{S}) = (\nabla \mathbf{v} + (\nabla \mathbf{v})^T)/3 - 2((\nabla \mathbf{v})^T : \mathbf{S})(\mathbf{S} + \mathbf{I}/3), \quad (8)$$

N is a dimensionless measure of the polymer number density c , $0 \leq \phi \leq 1$ is a nondimensional parameter related to the friction tensor, and λ is a relaxation time associated with the rotation of the rigid rods.

The degree of crystallization (θ_i) has been modeled by means of Ziabicki [2]

$$\frac{\partial \theta_i}{\partial t} + \mathbf{v} \cdot \nabla \theta_i = k_{Ai}(s)(\theta_{\infty, i} - \theta_i), \quad i = 1, 2, \quad (9)$$

where

$$k_{Ai}(s) = k_{Ai}(0) \exp(a_{2i} S_i^2) \quad (10)$$

is the linearized crystal growth rate, $k_{Ai}(0)$ is the amorphous growth rate, a_{2i} is a constant, and $\theta_{\infty, i}$ is the ultimate degree of crystallinity. The effective viscosity that appears in Eq. (4) can be written as

$$\mu_{eff} = G \exp(\eta(1 - T)) \cdot \exp(\beta(\theta/\theta_{\infty})^n) + \frac{2}{3} \alpha \lambda S^2, \quad (11)$$

where α is a parameter that relates the kinetic energy to the internal energy of the liquid-crystalline polymer, β is the crystallization viscosity rate and n is the crystallization viscosity index.

Results

In Figure 2, we show some sample results that illustrate the non-dimensional temperature field in the compound fiber as functions of the non-dimensional axial and radial coordinates. Figure 2 (a) corresponds to an average Biot number, Bi , of 5 and clearly indicates that a thermal boundary layer forms at R_2 and that the temperature of the core is initially almost uniform across. This figure also shows that the temperature is not uniform at the take-up point. As the Biot number increases, see Figure 2 (b), the heat transfer by conduction from the cladding increases, thereby increasing the heat flux at R_2 and decreasing the temperature of the core. In this case, the temperature at the take-up point is almost uniform.

As shown in Figure 3 (a), the average axial velocity exhibits a sigmoid shape characterized by a positive slope that levels off at about a non-dimensional distance from the nozzle equal to 0.5.

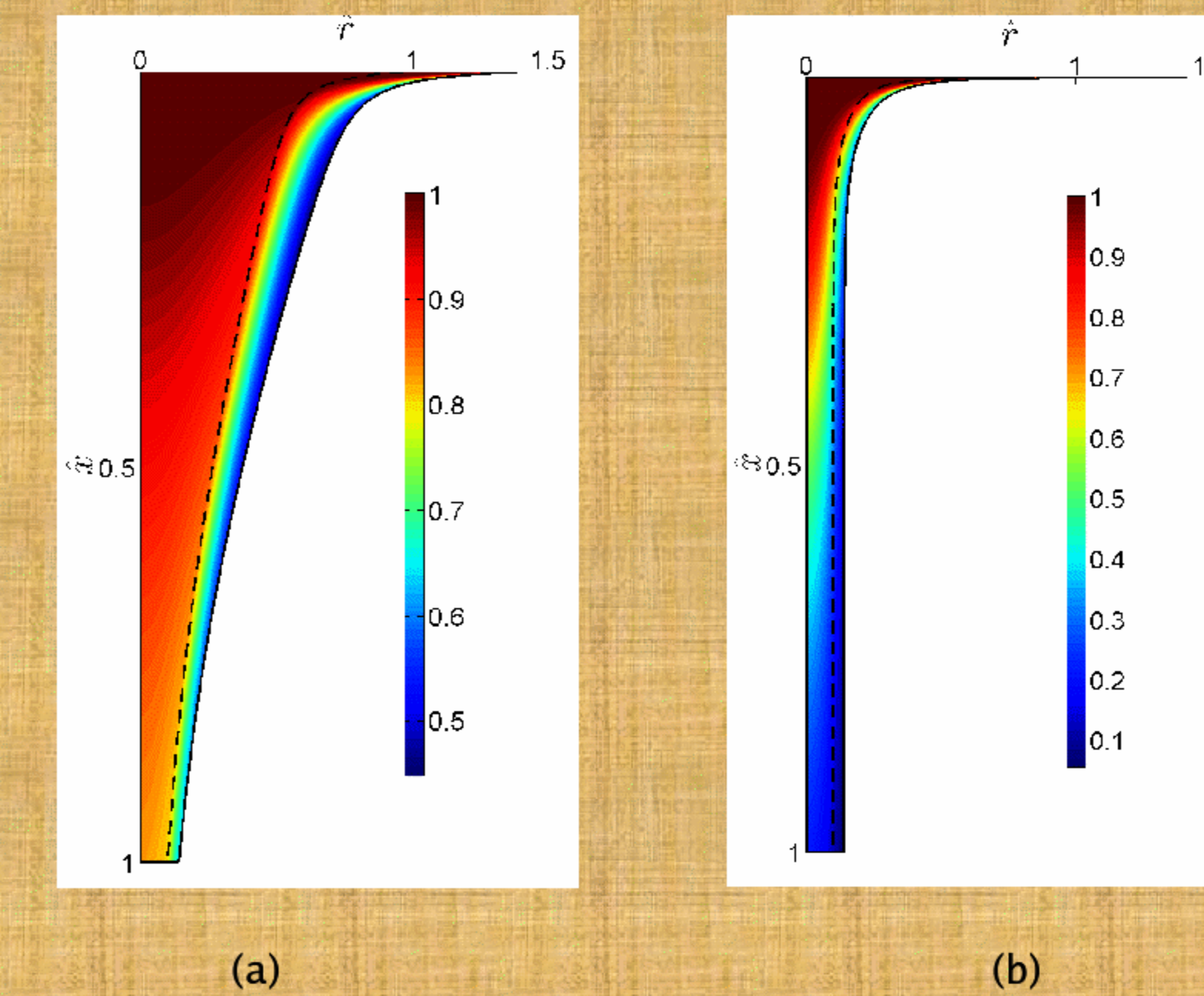


Figure 2. Temperature field of a compound fibre (a) $Bi = 5$ and (b) $Bi = 10$.

This figure also shows that the axial velocity component is uniform across the compound fiber, while the radial one is directed towards the symmetry axis at a rate that first increases and then decreases along the fiber, before it reaches a nil value upon crystallization.

For the conditions examined here, it has been observed that the molecular orientation parameter, see Figure 3 (e), defined as the square root of three-halves the contraction of the molecular orientation tensor with itself reaches a value equal to unity very close to the nozzle exit.

Figure 3 (f) shows that the degree of crystallization is mainly a function of the temperature. This is due to the rapid molecular orientation observed and the large increase in dynamic viscosity, see Figure 3 (d), of the compound liquid jet.

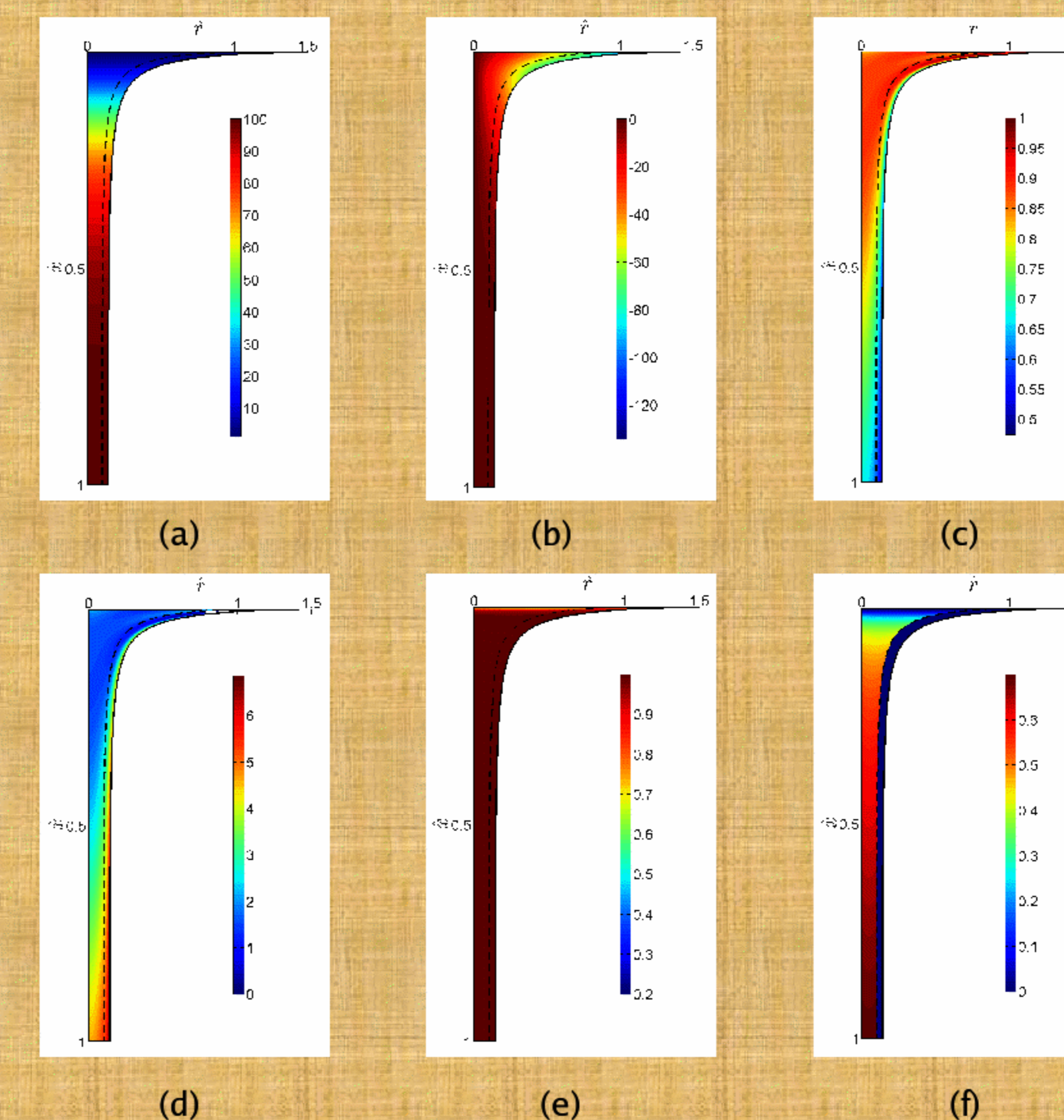


Figure 3. Two-dimensional axial velocity (a), radial velocity (b), temperature (c), (decimal logarithm of) the effective dynamic viscosity (d), molecular orientation parameter (e), and degree of crystallization (f) for a compound fiber.

Figure 4 (a) and (b) shows the degree of crystallization at transversal sections $x/L=0.01$ and 0.06 , respectively, for the core for a parabolic profile imposed to the molecular orientation parameter at $x = 0$. This figure clearly indicates that crystallization in the compound fiber for the conditions analyzed here is mainly a thermal process and that the flow-induced crystallization is small. Although, not shown here the degree of crystallization increases first rather quickly and then levels off.

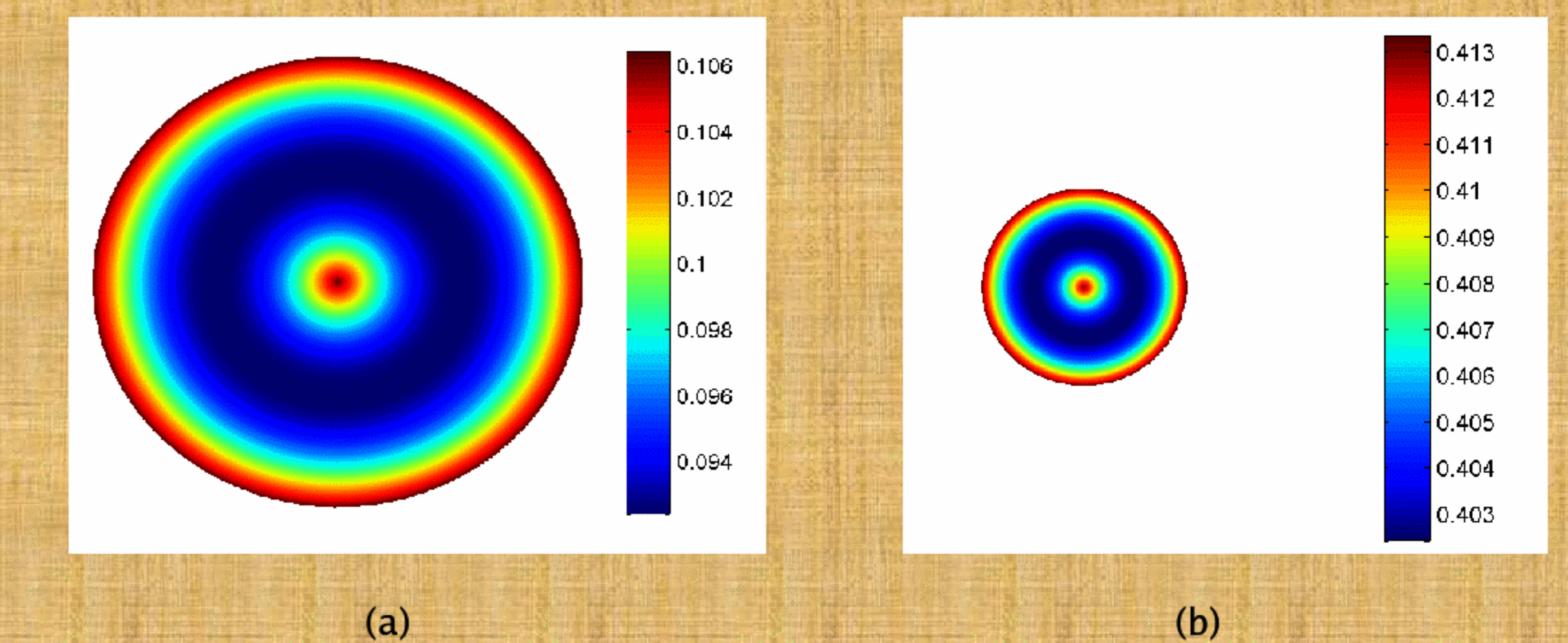


Figure 4. Degree of crystallization at (a) $x/L = 0.01$ and (b) $x/L = 0.06$.

Conclusions

A single-phase two-dimensional model of the spinning of compound plastic fibres that employs a Newtonian rheology modified by the degrees of crystallization and molecular orientation and temperature through an effective dynamic viscosity, and the molecular orientation of the liquid crystalline polymer through an orientation tensor that depends on the velocity field, has been proposed. For slender fibres and very low Biot numbers, an asymptotic analysis of the model yields one-dimensional equations for the leading-order axial velocity, temperature, orientation parameter and degree of crystallization provided that the molecular orientation is diagonal. For higher Biot numbers, the two-dimensional model was solved numerically, and its results indicate that substantial temperature non-uniformities in the radial direction exist even at large Biot numbers. These non-uniformities affect the degree of crystallization and have great effects on the optical properties of the compound fiber. For very slender fibres and small Biot numbers, good agreement between the leading-order one-dimensional model and the two-dimensional one has been observed. It was found that the crystallization of the compound fiber was mostly affected by thermal effects rather than by flow-induced ones.

Acknowledgments

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References

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Further Information

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